Potencial Coulombiano

$$V(r) = \frac{Q}{r} \Rightarrow \vec{E} = -\vec{\nabla} V = -\frac{\partial}{\partial r}\left(\frac{Q}{r}\right)\hat{r} \Rightarrow \vec{E} = \frac{Q}{r^2}\hat{r}$$

Função de Green e o Potencial

- $\nabla^2 G(\vec{r},\vec{r}\,') = \delta^3(\vec{r}-\vec{r}\,') \Leftrightarrow G(\vec{r},\vec{r}\,') = -4\pi\, G(\vec{r}-\vec{r}\,') = \frac{1}{|\vec{r}-\vec{r}\,'|}$

- $V(\vec{r}) = -\int_V G(\vec{r},\vec{r}\,')\,\rho(\vec{r}\,')\, d^3x = -\int_V \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\, d^3r'$

$$\Rightarrow \begin{cases} \nabla^2 V(\vec{r}) = -\rho(\vec{r}) \checkmark \\ \nabla^2 G(\vec{r},\vec{r}\,') = \delta^3(\vec{r}-\vec{r}\,') \end{cases} \Rightarrow \nabla^2 V(\vec{r}) = -\int_V \nabla^2 G(\vec{r},\vec{r}\,')\,\rho(\vec{r}\,')\, d^3x$$

$$\Rightarrow \nabla^2 V(\vec{r}) = -\int_V \delta^3(\vec{r}-\vec{r}\,')\,\rho(\vec{r}\,')\, d^3x \Rightarrow \nabla^2 V(\vec{r}) = -\rho(\vec{r}) \checkmark$$

Campo elétrico

$$\vec{E}(\vec{r}) = -\vec{\nabla}\cdot V(\vec{r}) = \vec{\nabla}\cdot\int_V \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\, d^3r' = -\int_V \rho(\vec{r}\,')\,\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\, d^3r'$$

Monopolo Magnetico [Problemas]

- $\vec{B} = \rho_m \cdot \frac{1}{r^2}\hat{r}$, onde ρ_m representa a densidade magnética de cargas

- $$\vec{B} = \vec{\nabla}_r \times \vec{A} = \overbrace{\left(\frac{1}{r\,\text{sen}\theta}\frac{\partial}{\partial\theta}(\text{sen}\theta\, A_\varphi) - \frac{\partial}{\partial\varphi}(A_\theta)\right)}^{0}\hat{r} + \overbrace{\left(\frac{1}{r}\left(\frac{1}{\text{sen}\theta}\frac{\partial (A_r)}{\partial\varphi} - \frac{\partial}{\partial r}(r A_\varphi)\right)\right)}^{0}\hat{\theta}$$
$$+ \underbrace{\left(\frac{1}{r}\left(\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial}{\partial\theta}(A_r)\right)\right)}_{0}\hat{\varphi}$$

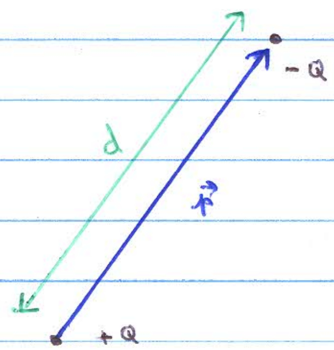
- $\vec{\nabla}_r\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\,\text{sen}\theta}\frac{\partial}{\partial\theta}(\text{sen}\theta\, A_\theta) + \frac{1}{r\,\text{sen}\theta}\frac{\partial A_\varphi}{\partial\varphi}$

$$\Rightarrow \vec{A} = \frac{\rho_m (1-\cos\theta)}{r\sin\theta}\,\hat{\varphi} \;\longrightarrow\; \begin{cases} \vec{A}_N = \rho_m \dfrac{(1-\cos\theta)}{r\sin\theta}\,\hat{\varphi} \\[2mm] \vec{A}_S = -\rho_m \dfrac{(1+\cos\theta)}{r\sin\theta}\,\hat{\varphi} \end{cases} \Rightarrow \vec{A}_S - \vec{A}_N = \frac{-2\rho_m}{r\sin\theta}\,\hat{\varphi} = \vec{\nabla}\Lambda$$

função de Calibre

singularidade em $\theta = \pi$ ⇒ não é globalmente definido

Dipolo



- $V = \dfrac{Q}{r} - \dfrac{Q}{|\vec{r}+\vec{d}|}$
- $f(\vec{r}+\vec{d}) \approx f(\vec{r}) + \vec{d}\cdot\nabla f(\vec{r})$

  $\Rightarrow \dfrac{1}{|\vec{r}+\vec{d}|} \approx \dfrac{1}{r} - \dfrac{\vec{d}\cdot\vec{r}}{r^3}$
- $\vec{p} = Q\vec{d}$

$$\Rightarrow V \approx \frac{Q}{r} - Q\left(\frac{1}{r} - \frac{\vec{d}\cdot\vec{r}}{r^3}\right) \Rightarrow \boxed{V_{dip} \simeq \frac{\vec{p}\cdot\vec{r}}{r^3}} \Rightarrow \boxed{\vec{E}_{dip} \approx \frac{2\vec{p}\cdot\vec{r}}{r^4}\,\hat{r}}$$

- fazendo $\vec{r}\,' \Leftrightarrow -\vec{d}$:

$$\Rightarrow V = -\int_V \frac{\rho(r')\,d^3r'}{|\vec{r}-\vec{r}\,'|} \longrightarrow V \approx \int_V \rho(r')\left(\frac{1}{r} + \frac{\vec{r}\cdot\vec{r}\,'}{r^3} + \dots\right) d^3r'$$

$$\Rightarrow V \simeq \frac{1}{r}\underbrace{\int_V \rho(r')\,d^3r'}_{Q} + \frac{1}{r^3}\underbrace{\int_V \rho(r')\,\vec{r}\,'\cdot\vec{r}\,d^3r'}_{\vec{p}} + \dots$$

$$\Rightarrow V \simeq \frac{Q}{r} + \frac{1}{r^3}\,\vec{p}\cdot\vec{r} + \text{expansões em multipolos}$$

OBS: $V_{quad} = \dfrac{r_i r_j Q_{ij}}{2r^3}$, $Q_{ij} = \displaystyle\int_V \rho(r')\left(3r'_i r'_j - \delta_{ij} r'^2\right) d^3r'$

/ /

Notação em D-branas

↳ formalismo em formas diferenciais

- n-forma : $A_{\mu_1 \dots \mu_n} = A_{(n)} = \frac{1}{n!} A_{\mu_1 \dots \mu_n} dx^{\mu_1} \wedge dx^{\mu_2} \wedge \dots \wedge dx^{\mu_n}$

- wedge ($\wedge$) : $dx^\mu \wedge dx^\nu = - dx^\nu \wedge dx^\mu$

- $\partial_\mu \rightarrow d = \frac{\partial}{\partial x^\mu} = dx^\mu$

- Dual de Hodge ($*$) : $*$ n-forma $\rightarrow$ (D-n)-forma

  ↳ $(*F)_{\mu_1 \dots \mu_{D-n}} = \frac{1}{n!} \varepsilon_{\mu_1 \dots \mu_{D-n}}^{\quad\nu_1 \dots \nu_n} F_{\nu_1 \dots \nu_n}$

- $\int_M F_{(n+1)} = \int_M F_{1 \dots n+1} \, d^{n+1}x$

- $F_{n+1} = dA_{(n)} = \frac{1}{n!} \left(\frac{\partial}{\partial x^\mu} A_{\mu_1 \dots} A_{\mu_n} \right) dx^{\mu_1} \wedge \dots \wedge dx^{\mu_n}$

Nas equações de Maxwell

$$\begin{cases} \partial_\mu \rightarrow d \\ \tilde{F} \rightarrow *F \end{cases} \Rightarrow \begin{cases} dF = *J_m \\ d(*F) = *J_e \end{cases}$$

Partículas puntiformes (carga: $\delta^3(\vec{x})$)

↳ Acoplamento : $e \int A_\mu \dot{x}^\mu d\tau = e \int A_\mu dx^\mu = e \int A_{(1)}$

Teorema

↳ não existe uma configuração de cargas em equilíbrio estável

$\Rightarrow$ As harmônicas não possuem máximos ou mínimos ($\nabla^2 V = 0$)

Energia Eletrostática em distribuicoes finitas

$$U(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{F}\cdot d\vec{r} = q\int_{\infty}^{\vec{r}} \vec{\nabla}V\cdot d\vec{r} \xrightarrow[V(\vec{r})\to 0]{\text{Se } r\to\infty} U(\vec{r}) = q\,V(\vec{r})$$

· Distribuicão Pontual

$$\begin{cases} U = \sum_i w_i = \dfrac{1}{2}\sum_{\substack{i,j \\ i\neq j}} \dfrac{q_i\, q_j}{|\vec{r}_i - \vec{r}_j|} \\ V(\vec{r}_i) \equiv \sum_{j\neq i} \dfrac{q_j}{|\vec{r}_i - \vec{r}_j|} \end{cases} \Rightarrow U = \frac{1}{2}\sum_i q_i\, V(\vec{r}_i)$$

OBS: resultado não inclui a autoenergia das partículas

· Distribuição contínua

$$\begin{cases} \sum_i \rightarrow \int d^3r \\ q_i \rightarrow \rho(\vec{r}) \end{cases} \Rightarrow U = \frac{1}{2}\int_{\mathbb{R}^3} \rho(\vec{r})\, V(\vec{r})\, d^3r$$

$$\cdot\begin{cases} \vec{\nabla}\cdot\vec{E} = 4\pi\,\rho(\vec{r}) \\ [V(\vec{r})]_{\partial\mathbb{R}^3} = 0 \\ \vec{\nabla}V = -\vec{E} \end{cases} \Rightarrow U = \frac{1}{8\pi}\int_{\mathbb{R}^3} E^2\, d^3r > 0$$

OBS: resultado já inclui a autoenergia das partículas

Exemplos

(I) Elétron de raio a na origem

$\cdot\ U = \dfrac{1}{8\pi}\displaystyle\int_{\mathbb{R}^3} E^2\, d^3r$

$\cdot\ \vec{E} = \dfrac{e}{r^2}\,\hat{r}$ $\Rightarrow U = \dfrac{1}{8\pi}\cdot 4\pi\displaystyle\int_a^{\infty} \dfrac{e^2}{r^2}\, dr \Rightarrow U = \dfrac{e^2}{2a} < m_e c^2$

$\cdot\ d^3r = 4\pi\, r^2\, dr$

/ /

III Entre dois dipolos

- $U = \frac{1}{2} \sum_i q_i V(\vec{r}_i)$

- $V_{dip} \simeq \frac{\vec{p} \cdot \vec{r}}{r^3}$ $\Rightarrow U \simeq \frac{Q}{2}\left(\underbrace{\frac{\vec{p}_1 \cdot \vec{r}}{r^3}}_{V(\vec{r})} - \underbrace{\frac{\vec{p}_1 (\vec{r} - \vec{d})}{r^3}}_{V(\vec{r}-\vec{d})} \right) \Rightarrow U \simeq \frac{\vec{p}_1 \cdot \vec{p}_2}{r^3}$

- $\vec{p} = Q\vec{d}$

OBS: interpretar o elétron como partícula singular falha para a região $r < a$. Da junção Relat. Restrita + Mec. Quântica surge:

$$\begin{cases} a = l_c = \frac{\hbar}{m_e c} \\ \dot{E} \neq 0 \end{cases}, \text{como } \frac{e^2}{2a} < m_e c^2 \Rightarrow \frac{\alpha}{2} < 1, \quad \alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$$

estrutura fina