/ /

- Solução Final $\longrightarrow$ $f(x) = \sum_n \frac{(-\nu)_n (\nu+1)_n}{(1)_n} \cdot \frac{1}{n!} \left(\frac{1-x}{2}\right)^n$

- Série Hipergeométrica $\longrightarrow$ $F(a,b,c,z) = \sum_n \frac{(a)_n (b)_n}{(c)_n} \cdot \frac{1}{n!} z^n$

Interpreta-se $f(x) = F(-\nu, \nu+1, 1, \frac{1-x}{2})$

Tomando $\nu = l \in \mathbb{Z}^+$ a solução é finita para $x = \pm 1$

$\Rightarrow$ $P_l(x) \equiv F(-l, l+1, 1, \frac{1-x}{2})$

- Se $m \neq 0$:

1) Calcular os expoentes ρ_1 e ρ_2

2) Encontrar $f(x) = (1-x)^{\frac{m}{2}} \sum_n c_n (1-x)^n$
   - Substituir na eq. de Laplace $\neq 0$
   - Achar relações de recursão do c_n

3) $P(x) = (1-x^2)^{\frac{m}{2}} F(-\nu+m, \nu+m+1, 1+m, \frac{1-x}{2})$

4) Impor regularidade em $x = -1$ com $m-\nu \in \mathbb{Z}^+$ $\Rightarrow$ $\nu = l \geq m$

5) $P_{lm}(x) = c_{lm} (1-x^2)^{\frac{m}{2}} F(m-l, m+l+1, m+1, \frac{1-x}{2})$
   - $l = 0, 1, 2 \ldots$
   - $m = -l, \ldots 0, \ldots, l$

   ↳ Função Associada de Legendre

6) $P(\theta) Q(\varphi) \equiv Y_l^m(\theta, \varphi) = c_{lm} P_{lm}(\cos\theta) e^{im\varphi}$

   ↳ Harmônica esférica

Para (II):

$\frac{U(r)}{r} \longrightarrow U(r) = A r^{l+1} + B r^{-l}$

tilibra

Transformação Geral de Coordenadas (TGC)

↳ Assemelha-se à uma rotação mas depende dos pontos transformados "$T(x)$"

- Elemento de linha: $ds^2 = dx^2 + dy^2 + dz^2 = \underbrace{g_{ij}}_{\text{métrica}} dx^i dx^j$

- Linearidade infinitesimal: $dx^i = \dfrac{\partial x^i}{\partial x^j} dx^j$ → $S^i{}_j(x) = \dfrac{\partial x'^i}{\partial x^j}$; $(S^{-1})^i{}_j(x) = \dfrac{\partial x^i}{\partial x'^j}$

- Invariância do ds^2:

$$ds^2 = ds'^2 \Rightarrow g_{ij}(x)\, dx^i dx^j = g'_{mn}(x)\, dx'^m dx'^n$$

$$\Rightarrow g_{ij}(x) \frac{\partial x^i}{\partial x'^m} \frac{\partial x^j}{\partial x'^n} dx'^m dx'^n = g'_{mn}(x)\, dx'^m dx'^n$$

$$\Leftrightarrow g'_{mn}(x') = g_{ij}(x) \frac{\partial x^i}{\partial x'^m} \frac{\partial x^j}{\partial x'^n}$$

- Classificação sob TGC
  - Vetor contravariante → $w'^i(x') = S^i{}_j(x)\, w^j(x)$
  - Vetor covariante → $w'_i(x) = w_j(x')\, (S^{-1})^j{}_i(x)$
  - Escalar → $\phi'(x') = \phi(x)$

- Área e Volume com métrica invariante

$$\underbrace{\det\left[g'_{mn}(x')\right]}_{g'} = \det\left[g_{ij}(x) \frac{\partial x^i}{\partial x'^m} \frac{\partial x^j}{\partial x'^n}\right] = \underbrace{\det\left[g_{ij}(x)\right]}_{g} \cdot \underbrace{\det\left[\frac{\partial x^i}{\partial x'^m}\right]}_{J} \cdot \underbrace{\det\left[\frac{\partial x^j}{\partial x'^n}\right]}_{J}$$

$$g' \longrightarrow \boxed{g' = g\, J^2} \longleftarrow g$$

$$\Rightarrow (d^3x)(\sqrt{g}) \rightarrow (d^3x'\, J)\left(\frac{\sqrt{g'}}{\sqrt{J^2}}\right) = d^3x' \sqrt{g'}$$

↳ nova métrica Invariante

Derivada covariante

- Divergente:

$\Rightarrow$ defina $S = \int \underbrace{d^3x \sqrt{g}}_{\text{escalar}} \underbrace{w^i(x) \underbrace{\partial_i \phi(x)}_{\text{vetor}}}_{\text{escalar}}$ $\longrightarrow$ partes $\rightarrow$

escalar

$$\rightarrow \quad S = -\int d^3x\, \partial_i(\sqrt{g}\, w^i(x))\, \phi(x) + \underbrace{\text{termo de borda}}_{\to 0}$$

$$\Rightarrow \quad S = -\int \underbrace{d^3x \sqrt{g}}_{\text{escalar}} \cdot \frac{1}{\sqrt{g}} \partial_i(\sqrt{g}\, w^i(x))\, \phi(x)$$

$\hookrightarrow$ escalar

$$\Leftrightarrow \quad \boxed{D_i w^i \equiv \frac{1}{\sqrt{g}} \partial_i(\sqrt{g}\, w^i)} \quad \text{é um escalar}$$

- Laplaciano:

$\Rightarrow$ defina $S = \int \underbrace{d^3x \sqrt{g}}_{\text{escalar}} \overbrace{g^{ij}(x)\, \partial_i \phi(x)\, \partial_j \phi(x)}^{\text{escalar}}$ $\longrightarrow$ partes $\rightarrow$

$\hookrightarrow$ vetor

$$\rightarrow \quad S = -\int d^3x\, \partial_i(\sqrt{g}\, g^{ij}(x)\, \partial_j \phi(x)) \cdot \phi(x) + \underbrace{\text{termo de borda}}_{\to 0}$$

$$\Rightarrow \quad S = -\int \underbrace{d^3x \sqrt{g}}_{\text{escalar}} \cdot \frac{1}{\sqrt{g}} \partial_i(\sqrt{g}\, g^{ij}(x)\, \partial_i \phi(x)) \cdot \phi(x)$$

escalar

$$\Leftrightarrow \quad \boxed{D^2\phi \equiv \frac{1}{\sqrt{g}} \partial_i(\sqrt{g}\, g^{ij}(x)\, \partial_j \phi(x))} \quad \text{é um escalar}$$