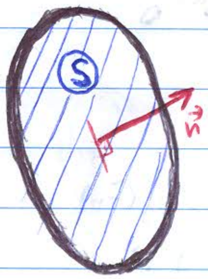
Se $\vec{J}$ restringe-se a um contorno $\mathcal{C}$ "fio":

$$I = \int_S \vec{J}\cdot d\vec{s} \simeq J\int_S ds = J\cdot A$$

$$\vec{J}\,dV \simeq JA\,d\vec{x} \rightarrow I\,d\vec{x} \qquad \Rightarrow \vec{B} = \oint_{\mathcal{C}} \frac{d\vec{x}\times(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$$

Dipolos Magnéticos



$$A(\vec{r}) = \int_V d^3r' \frac{J(r')}{|\vec{r}-\vec{r}'|} = I\oint_C \frac{1}{|\vec{r}-\vec{r}'|}\,dr' \rightarrow \text{Taylor} \rightarrow$$

$$\simeq I\oint_C dr'\left(\underbrace{\frac{1}{r}}_{\text{termo de borda}\,\to\,0} + \frac{\vec{r}\cdot\vec{r}'}{r^3} + \dots\right) \simeq I\oint_C \frac{\vec{r}\cdot\vec{r}'}{r^3}\,dr'$$

OBS:

Seja $\vec{g}$ um vetor constante

$$\Rightarrow \oint \vec{g}\,(\vec{r}\cdot\vec{r}')\,d\vec{r}' \xrightarrow{\text{Stokes}} \int_{S'} d\vec{S}'\cdot\vec{\nabla}_{r'}\times\left(\vec{g}\,(\vec{r}\cdot\vec{r}')\right)$$, nas componentes

$$\Rightarrow \int_{S'} dS'_i\,\varepsilon_{ijk}\,\partial'_j\,(g_k r_m r'_m) = \int_{S'} dS'_i\,\varepsilon_{ijk}\,g_k r_m \delta_{jm} = \int_{S'} dS'_i\,\varepsilon_{ijk}\,g_k r_j$$

$$= g_k \int_{S'} \varepsilon_{kij}\,dS'_i\,r_j \Leftrightarrow \vec{g}\cdot\int_{S'} d\vec{S}'\times\vec{r} = \vec{g}\cdot\vec{S}\times\vec{r}$$,

$\int_{S'} d\vec{S}'$ → vetor Área → $\vec{S}$

Tomando $\vec{g}$ como a identidade e substituindo o resultado em $\vec{A}$:

$$\Rightarrow \vec{A}(\vec{r}) \simeq \frac{I}{r^3}\oint_C \vec{r}\cdot\vec{r}'\,d\vec{r}' \longrightarrow \vec{A}(\vec{r}) \simeq \frac{I\,\vec{S}\times\vec{r}}{r^3}$$

$$\Leftrightarrow \vec{A}(\vec{r}) \simeq \frac{\vec{m}\times\vec{r}}{r^3}, \quad \vec{m} \equiv I\vec{S}$$ "momento de dipolo magnético"

$$\Rightarrow \vec{B} = \vec{\nabla}\times\vec{A}(\vec{r}) = \frac{3(\vec{m}\cdot\hat{r})\,\hat{r} - \vec{m}}{r^3}$$

Distribuição geral de Correntes

$$A_i(\vec{r}) = \int_V d^3r' \frac{J_i(\vec{r}')}{|\vec{r}-\vec{r}'|} \simeq \int_V d^3r' \left(\frac{J_i(\vec{r}')}{r} + \frac{J_i(\vec{r}')\,\vec{r}\cdot\vec{r}'}{r^3} + \dots \right)$$

• Primeiro Termo:

$$\partial_j(J_j r'_i) = \underbrace{\partial_j(J_j)}_{\text{corrente estacionária} \to 0} r'_i + J_j \partial_j(r'_i) = J_j \delta_{ij} = J_i$$

$$\Rightarrow \int_V d^3r' \frac{J_i(\vec{r}')}{r} = \frac{1}{r}\int_V d^3r' \partial_j(J_j r'_i) \xrightarrow{\text{Termo de borda}} 0$$

• Segundo Termo:

• $\partial_j(J_j r'_i r'_k) = \partial_j(J_j) r'_i r'_k + J_j \partial_j(r'_i) r'_k + J_j r'_i \partial_j(r'_k)$

$= J_j \delta_{ji} r'_k + J_j r'_i \delta_{jk} = J_i r'_k + J_k r'_i$

• $J_i \vec{r}\cdot\vec{r}' = \frac{1}{2} J_i r_k r'_k + \frac{1}{2} J_i r_k r'_k = \frac{1}{2} r_k J_i r'_k + \frac{1}{2} r_k J_i r'_k =$

$= \frac{1}{2} J_i r_k r'_k + \frac{1}{2} r_k \left(\partial_j(J_j r'_i r'_k) - J_k r'_i\right)$

$= \frac{1}{2} J_i r_k r'_k - \frac{1}{2} J_k r_k r'_i + \frac{1}{2} r_k \partial_j(J_j r'_i r'_k)$

$$\Rightarrow \int_V d^3r' \frac{J_i(\vec{r}')\,\vec{r}\cdot\vec{r}'}{r^3} = \frac{1}{r^3}\left[\int_V d^3r' \frac{1}{2} r_k (J_i r'_k - J_k r'_i) + \underbrace{\int_V d^3r' \frac{1}{2} r_k \partial_j(J_j r'_i r'_k)}_{\text{borda} \to 0}\right]$$

$$= \frac{1}{r^3}\int_V \frac{d^3r'}{2}\left(\underbrace{\vec{J}(\vec{r}\cdot\vec{r}') - \vec{r}'(\vec{r}\cdot\vec{J})}_{\vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})}\right) = \frac{1}{2r^3}\int_V d^3r' \underbrace{\vec{r}\times(\vec{J}\times\vec{r}')}_{\vec{a}\times(\vec{b}\times\vec{c})}$$

$$\Rightarrow \vec{A}(\vec{r}) \simeq \frac{1}{2r^3}\vec{r}\times\int_V d^3r' \vec{J}\times\vec{r}'$$

$$\Leftrightarrow \vec{A}(\vec{r}) \simeq \frac{\vec{m}\times\vec{r}}{r^3}, \quad \vec{m} \equiv \frac{1}{2}\int_V d^3r' (\vec{r}'\times\vec{J})$$

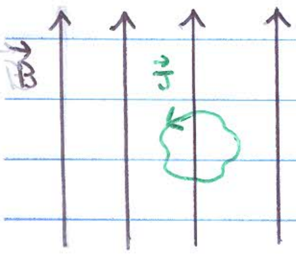
Força Magnética: Caso geral com $\vec{J}_1$ e $\vec{J}_2$

$$\vec{B}_1(\vec{r}) = \int_V d^3r' \, \frac{\vec{J}_1(\vec{r}') \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|} \Rightarrow \vec{F}_M = \int_V \vec{J}_2(\vec{r}) \times \vec{B}_1(\vec{r}) \, d^3r$$

• Com $\vec{J}_1$ e $\vec{J}_2$ restritos a contornos C_1 e C_2 "fios":

$$\Rightarrow \vec{F}_M = I_1 I_2 \oint_{C_1} \oint_{C_2} d\vec{r}_2 \times \left(d\vec{r}_1 \times \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3} \right)$$

• Aproximação em energia com $\vec{B}$ de fundo:



• $\vec{F} = \int_V d^3r \, \vec{J}(\vec{r}) \times \vec{B}(\vec{r})$ → $\vec{J}$ próximo à $\vec{r} \approx \vec{R}$ (CM)
→ $\vec{B}$ varia pouco próximo à $\vec{r} \approx \vec{R}$

• $\vec{B}(\vec{r}) \approx \vec{B}(\vec{R}) + (\vec{r} \cdot \vec{\nabla}) \vec{B}(\vec{R})$

$$\Rightarrow \vec{F} \simeq \int_V d^3r \, \vec{J}(\vec{r}) \times \left(\vec{B}(\vec{R}) + (\vec{r} \cdot \vec{\nabla}) \vec{B}(\vec{R}) \right)$$

corrente fechada → 0

$$\Rightarrow \vec{F} \simeq - \vec{B}(\vec{R}) \times \int_V d^3r \, \vec{J}(\vec{r}) + \int_V d^3r \, \vec{J}(\vec{r}) \times \left((\vec{r} \cdot \vec{\nabla}) \vec{B}(\vec{R}) \right)$$

OBS: como $\vec{\nabla} \times \vec{B} = 0 \Rightarrow \varepsilon_{mlk} \partial_l B_k = \partial_l B_m - \partial_k B_l = 0$

$\Rightarrow \varepsilon_{ijk} J_i r_l (\partial_l B_k - \partial_k B_l) = 0$

$\Rightarrow \varepsilon_{ijk} J_i r_l \partial_l B_k = \varepsilon_{ijk} J_i r_l \partial_k B_l$

$\Rightarrow -\varepsilon_{jik} J_i B_k r_l \partial_l = \varepsilon_{jki} \partial_k J_i r_l B_l$

$\Leftrightarrow -\vec{J} \times \vec{B} (\vec{r} \cdot \vec{\nabla}) = \vec{\nabla} \times \vec{J} (\vec{r} \cdot \vec{B}) \Leftrightarrow \vec{J} \times ((\vec{r} \cdot \vec{\nabla}) \vec{B}) = - \vec{\nabla} \times ((\vec{r} \cdot \vec{B}) \vec{J})$

daí: $\vec{F} \simeq - \vec{\nabla} \times \int_V d^3r \left((\vec{r} \cdot \vec{B}(\vec{R})) \vec{J}(\vec{r}) \right)$

$$\Rightarrow \vec{F} \simeq - \vec{\nabla} \times \left(\vec{B} \times \overbrace{\int_V d^3r \, \frac{\vec{J} \times \vec{r}}{2}}^{-\vec{m}} \right) = + \vec{\nabla} \times (\vec{B} \times \vec{m}) = \vec{B} \times (\vec{\nabla} \times \vec{m})$$

$$\Rightarrow \vec{F} \simeq \vec{B} \times (\vec{\nabla} \times \vec{m}) = \vec{\nabla}(\vec{B} \cdot \vec{m}) - \vec{m} \underbrace{(\vec{\nabla} \cdot \vec{B})}_{\text{nulo}}$$

$$\Rightarrow \vec{F} \simeq \vec{\nabla}(\vec{B} \cdot \vec{m}) = - \vec{\nabla} U \Leftrightarrow U = - \vec{B} \cdot \vec{m}$$