Relatividade Restrita

1)

Observer O' is travelling with velocity $v = 0.6c$ in the x direction relative to observer O. Each observer has a meter stick with one end fixed at his origin and the other end fixed at x (or x') $= 1$ m (see Figure P.2.1). Each stick has a marking device (such as a spring-loaded pin) at the high x (or

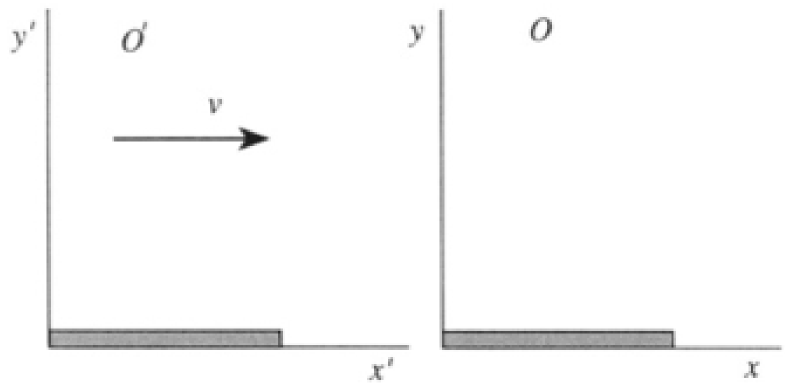

Figure P.2.1

x') end, capable of marking the other stick if it overlaps that stick when the marking devices are triggered. The two origins coincide at $t = t' = 0$. Both marking devices are triggered at $t = 0$.

a) According to O, who has the shorter stick? Which stick is marked and where?

b) According to O', who has the shorter stick? Prove by explicit derivation that O' agrees on the result of the marking experiment, including the position of the mark.

2)

A person on Earth observes two rocket ships moving directly toward each other and colliding as shown in Figure P.2.2a. At time $t = 0$ in the Earth

Figure **P.2.2a**

frame, the Earth observer determines that rocket A, travelling to the right at $v_A = 0.8c$, is at point a, and rocket B is at point b, travelling to the left at $v_B = 0.6c$. They are separated by a distance $l = 4.2 \cdot 10^8$ m (see Figure P.2.2b).

Figure **P.2.2b**

a) In the Earth frame, how much time will pass before the rockets collide?
b) How fast is rocket B approaching in A's frame? How fast is rocket A approaching in B's frame?
c) How much time will elapse in A's frame from the time rocket A passes point a until collision? How much time will elapse in B's frame from the time rocket B passes point b until collision?

3)

a) Consider two successive Lorentz transformations of the three frames of reference K_0, K_1, K_2. K_1 moves parallel to the x axis of K_0 with velocity v, as does K_2 with respect to K_1. Given an object moving in the x direction with velocity v_2 in K_2, derive the formula for the transformation of its velocity from K_2 to K_0.
b) Now consider $n+1$ frames moving with the same velocity v relative to one another (see Figure P.2.6). Derive the formula for a Lorentz transformation from K_n to K_0, if the velocity of the object in K_n is also v.

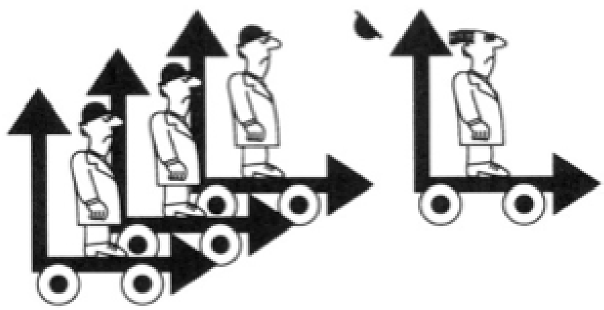

Figure P.2.6

Hint: You may want to use the definition of rapidity or velocity parameter, $\tanh\psi = \beta$, where $\beta = v/c$.

4)

A rocket having initially a total mass M_0 ejects its fuel with constant velocity $-u$ $(u > 0)$ relative to its instantaneous rest frame. According to Newtonian mechanics, its velocity V, relative to the inertial frame in which it was originally at rest, is related to its mass $M(V)$ by the formula

$$\frac{M}{M_0} = \exp\left(-\frac{V}{u}\right)$$

a) Derive this result.
b) Suppose the velocity of the ejecta is limited only by $0 \le u \le c$, and derive the relativistic analogue of the above equation. Show that it reduces to the Newtonian result at the appropriate limit.