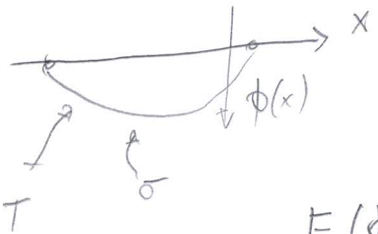
16/3/2016

$$S[q(t),\dot q(t)] = \int_{t_0}^{t_f} dt\, L[q(t),\dot q(t)]$$



$$\boxed{T\frac{d^2\phi(x)}{dx^2} = -\sigma g}$$

$$\boxed{m\ddot q(t) = -mg}$$

$$E(\phi) = \int dx \left[\frac{T}{2}\left(\frac{d\phi}{dx}\right)^2 - \sigma g \phi\right] \rightarrow \boxed{\delta E(\phi) = 0}$$

$$S = \int dt\, \underbrace{\left[\frac{m}{2}\dot q^2 - mgq\right]}_{L}$$

$$= \int_{t_0}^{t_f} dt\, \underbrace{\left[\frac{m}{2}\dot q^2 - V(q)\right]}_{L}$$

$$\boxed{m\ddot q = -\frac{dV}{dq}}$$

$$q(t) \longrightarrow q(t) + \delta q(t)\ , \quad \delta q(t_0) = 0 = \delta q(t_f)$$

$$S[q+\delta q, \dot q + \delta \dot q] = \int dt\, L[q+\delta q, \dot q + \delta \dot q] =$$

$$\boxed{\delta S \equiv S' - S = 0}$$

$$= \int dt \left\{ L[q,\dot q] + \frac{\partial L}{\partial q}\cdot \delta q + \frac{\partial L}{\partial \dot q}\delta \dot q \right\}$$

$$\delta S = \int dt \left\{ \frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot q}\delta \dot q \right\} = 0 \quad \forall\, \delta q(t)$$

$$\boxed{\delta \dot q = \frac{d}{dt}(\delta q)}$$

$$= \int dt \left\{ \frac{\partial L}{\partial q}\delta q + \frac{d}{dt}\left[\frac{\partial L}{\partial \dot q}\delta q\right] - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\delta q\right)\right\}$$

$$= \int dt\, \delta q \left\{ \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\right\} + \cancel{\left[\frac{\partial L}{\partial \dot q}\delta q\right]_{t_0}^{t_f}}$$

$$\therefore \boxed{\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0} \longrightarrow \underline{\text{Euler - Lagrange}}$$

$$L = \frac{m\dot{q}^2}{2} - V(q) \Rightarrow \begin{cases} \dfrac{\partial L}{\partial q} = -\dfrac{dV}{dq} \\[2ex] \dfrac{\partial L}{\partial \dot{q}} = m\dot{q} \end{cases}$$

$$\Rightarrow -V' - m\ddot{q} = 0 \rightarrow \boxed{m\ddot{q} = -\frac{dV}{dq}}$$

$$L = \frac{m}{2}\dot{q}^2 - V(|\vec{q}|) \longrightarrow \dot{\vec{q}} \cdot \dot{\vec{q}} = g_{\mu\nu}\dot{q}^\mu \dot{q}^\nu$$

para um espaço generalizado

$$= \frac{m}{2} g_{\mu\nu}\dot{q}^\mu \dot{q}^\nu - V(|\vec{q}|)$$

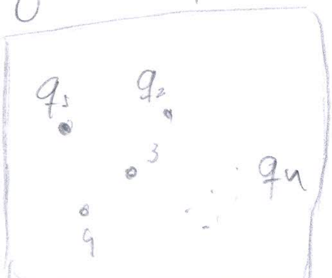
$$\boxed{\nabla^2 \phi = 4\pi G \rho}$$

potencial gravitacional

Newton

$$E(\phi) = \int d^3x \left[\frac{1}{8\pi G}(\vec{\nabla}\phi)^2 + \rho\phi\right]$$

$$S_{grav.} = -\int dt \int d^3x \left[\frac{1}{8\pi G}(\vec{\nabla}\phi)^2 + \rho\phi\right] =$$



$$\boxed{\rho(\vec{x}) = \sum_{a=1}^{N} m_a \delta^3(\vec{x} - q_a(t))}$$

$$= -\int dt \int d^3x \frac{1}{8\pi G}(\vec{\nabla}\phi)^2 - \underbrace{\int dt \int d^3x\, \phi(\vec{x}) \sum_a m_a \delta^3(\vec{x} - \vec{q}_a(t))}$$

$$\int dt \sum_a \underbrace{m_a \phi(\vec{q}_a(t))}_{V_a(q_a(t))}$$

$$S = -\int dt \int d^3x \left[\frac{1}{8\pi G}(\vec{\nabla}\phi) + \rho\phi\right]$$

$$+ \int dt \sum_a \frac{1}{2} m_a \dot{q}_a^2$$

$$L = \frac{m}{2}\dot{q}^2 - V$$

$$H = p\dot{q} - L \qquad p \equiv \frac{\partial L}{\partial \dot{q}} = m\dot{q}$$

$$= \frac{p^2}{m} - \frac{p^2}{2m} + V = \text{energia}$$

$$\longrightarrow \frac{\partial H}{\partial q} \overset{?}{=} -\dot{p} \qquad \frac{\partial H}{\partial p} = \dot{q}$$