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Resolução da lista

lista 2 - Ex 1)

$$\begin{cases} g_{\mu\nu}(x) = \delta_{\mu\nu} + A_{\mu\nu\lambda} x^{\lambda} + \dots & (1) \\ x^{\mu} = x'^{\mu} + L^{\mu}{}_{\nu\lambda} x'^{\nu} x'^{\lambda} + \dots & (2) \end{cases}$$

$$g'_{\rho\sigma}(x) = \frac{\partial x^{\mu}}{\partial x'^{\rho}} \frac{\partial x^{\nu}}{\partial x'^{\sigma}} g_{\mu\nu}(x) \quad (3)$$

(2): $$\frac{\partial x'^{\mu}}{\partial x'^{\rho}} = \frac{\partial x'^{\mu}}{\partial x^{\rho}} + 2L^{\mu}{}_{\nu\lambda} \frac{\partial x'^{\nu}}{\partial x'^{\rho}} x'^{\lambda} + \dots$$

$$= \delta^{\mu}{}_{\rho} + 2L^{\mu}{}_{\rho\lambda} x'^{\lambda} + \dots$$

$\mu \leftrightarrow \nu$, $\rho \leftrightarrow \sigma$: $$\frac{\partial x^{\nu}}{\partial x'^{\sigma}} = \delta^{\nu}{}_{\sigma} + 2L^{\nu}{}_{\sigma\lambda} x^{\lambda} + \dots$$

derivando a (2)

Em (3):

$$g'_{\rho\sigma}(x') = (\delta^{\mu}_{\rho} + 2L^{\mu}{}_{\rho\lambda} x'^{\lambda} + \dots)(\delta^{\nu}{}_{\sigma} + 2L^{\nu}{}_{\sigma\alpha} x'^{\alpha} + \dots) g_{\mu\nu}(x)$$

$$= (\delta^{\mu}_{\rho} \delta^{\nu}_{\sigma} + 2\delta^{\mu}_{\rho} L^{\nu}{}_{\sigma\alpha} x'^{\alpha} + 2\delta^{\nu}_{\sigma} L^{\mu}{}_{\rho\lambda} x'^{\lambda} + \dots) g_{\mu\nu}(x) \quad (4)$$

(2) em (1): $$g_{\mu\nu}(x) = \delta_{\mu\nu} + A_{\mu\nu\lambda}(x'^{\lambda} + L^{\lambda}{}_{\alpha\beta} x'^{\alpha} x'^{\beta} + \dots) \quad (5)$$

(5) em (4): $$g'_{\rho\sigma}(x') = (\delta^{\mu}_{\rho} \delta^{\mu}_{\sigma} + 2\delta^{\mu}_{\rho} L^{\nu}{}_{\sigma\alpha} x'^{\alpha} + 2\delta^{\mu}_{\sigma} L^{\mu}{}_{\rho\lambda} x'^{\lambda} + \dots)$$

$$\cdot (\delta_{\mu\nu} + A_{\mu\nu\tau}(x'^{\tau} + L^{\tau}{}_{\xi\beta} x'^{\xi} x'^{\beta})$$

Ordem $(x')^0$:

$$\delta^\mu_\rho \delta^\nu_\sigma \delta_{\mu\nu} = \delta^\mu_\rho \delta^\mu_\sigma = \delta_{\rho\sigma} \quad ✓$$

Ordem $(x')^1$:

$$x^\lambda\left(\delta^\mu_\rho \delta^\nu_\sigma A_{\mu\nu\lambda} + 2\delta_{\mu\nu}\delta^\mu_\rho L'^\nu_{\sigma\lambda} + 2\delta_{\mu\nu}\delta^\nu_\sigma L'^\mu_{\rho\lambda}\right) = 0$$

⊛ $A_{\rho\sigma\lambda} + 2\delta_{\rho\mu} L^\mu_{\sigma\lambda} + 2\delta_{\mu\sigma} L^\mu_{\rho\lambda} = 0$

Ciclicamente

$\rho \to \sigma \to \lambda \to \rho$

⊛⊛ $A_{\sigma\lambda\rho} + 2\delta_{\sigma\mu} L^\mu_{\lambda\rho} + 2\delta_{\mu\lambda} L^\mu_{\sigma\rho} = 0$

⊛⊛⊛ $A_{\lambda\rho\sigma} + 2\delta_{\lambda\mu} L^\mu_{\rho\sigma} + 2\delta_{\mu\rho} L^\mu_{\lambda\sigma} = 0$

⊛ + ⊛⊛ − ⊛⊛⊛

$\delta_{\mu\nu} = \delta_{\nu\mu}$

$L^\mu_{\alpha\rho} = L^\mu_{\rho\alpha}$

$$\left[A_{\rho\sigma\lambda} + A_{\sigma\lambda\rho} - A_{\lambda\rho\sigma} = -4\delta_{\mu\sigma} L^\mu_{\lambda\rho}\right]\delta^{\nu\sigma}$$

<u>RHS</u>: $-4\,\delta^{\nu\sigma}\delta_{\mu\sigma} L^\mu_{\lambda\rho} = -4L^\nu_{\lambda\rho}$

<u>LHS</u>: $A_{\rho\nu\lambda} + A_{\nu\lambda\rho} - A_{\lambda\rho\nu}$

$$L^{\nu}{}_{\lambda\rho} = \frac{1}{4}\, \delta^{\nu\sigma}\left(A_{\lambda\rho\sigma} - A_{\rho\sigma\lambda} - A_{\sigma\lambda\rho}\right)$$

3) $g_{\mu\nu} = a_\mu \delta_{\mu\nu}$ (não tem soma sobre μ)

$g_{\mu\mu} \neq 0 \quad g^{\mu\mu} = \frac{1}{g_{\mu\mu}}$ (não tem soma sobre μ)

~~5)~~ $\cancel{\Gamma'^{\lambda}{}_{\mu\nu} = \frac{\partial x'^{\lambda}}{\partial x^{\alpha}} \frac{\partial x^{\omega}}{\partial x'^{\mu}}}$

Lista 1 : 1) $\phi(x)$ escalar : $\phi'(x') = \phi(x)$

$$(\nabla\phi \cdot \nabla\phi) \to \nabla'\phi' \cdot \nabla'\phi' = \sum_{i=1} \frac{\partial}{\partial x_i}\phi \, \frac{\partial \phi}{\partial x'^i}$$

$$= R^{ki} \frac{\partial \Phi}{\partial x^i} \; R^{kj} \frac{\partial \phi}{\partial x^j} = (R^{ik})^T R^{kj} \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_j}$$

$$\nabla^2 \phi = \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} \phi \to R^{ki} R^{kj} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} \Phi = \delta^{ij} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} \Phi = \nabla^2 \Phi$$

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3.) $(\theta, \varphi) : \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix} \to (x, y) : g'_{\rho\sigma}(x) = g_{\mu\nu}(x) \frac{\partial x^\mu}{\partial x'^\rho} \frac{\partial x^\nu}{\partial x'^\sigma}$

$$\begin{cases} x = \frac{W}{2\pi} \varphi \\ y = -\frac{W}{2\pi} \log\left(\operatorname{tg} \frac{\theta}{2}\right) \end{cases}$$

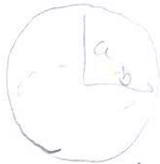
$$\begin{cases} g'_{xx} = \cancel{\left(\frac{\partial \theta}{\partial x}\right)^2} + \sin^2\theta \left(\frac{\partial \varphi}{\partial x}\right)^2 \\ g'_{xy} = \cancel{\left(\frac{\partial \theta}{\partial x}\right)}\left(\frac{\partial \theta}{\partial y}\right) + \sin^2\theta \cancel{\left(\frac{\partial \varphi}{\partial x}\right)\left(\frac{\partial \varphi}{\partial y}\right)} = 0 \end{cases}$$

$$g'_{yy} = \left(\frac{\partial \theta}{\partial y}\right)^2 + \sin^2\theta \left(\frac{\partial \varphi}{\partial y}\right)^2$$

$$g'_{\mu\nu} = \begin{pmatrix} \sin^2\theta \left(\frac{\partial \varphi}{\partial x}\right)^2 & 0 \\ 0 & \left(\frac{\partial \theta}{\partial y}\right)^2 \end{pmatrix}$$

$\varphi = \frac{2\pi}{W} x \quad \theta = 2\operatorname{tg}^{-1} \exp\left(\frac{-2\pi y}{W}\right)$

$$= \Omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \Omega^2 = \frac{4\pi^2}{W^2} \frac{1}{\cosh^2\left(\frac{2\pi y}{W}\right)}$$

12)  Equador

$a \neq b$

$$ds^2 = (b^2 + a^2\cos^2\theta)\, d\theta^2 + \frac{(b^2+a^2)^2}{b^2 + a^2\cos^2\theta}\sin^2\theta\, d\varphi^2$$

$\theta = \frac{\pi}{2} \rightarrow ds^2 = \frac{(b^2+a^2)^2}{b^2}\, 1\, d\varphi^2$

$\rightarrow d\theta = 0$

$$\int_0^{2\pi} d\varphi\, \frac{b^2+a^2}{b} = 2\pi\,\frac{b^2+a^2}{b} \overset{a=b=R}{=} 2\pi R$$

longitude

$\varphi = cte$

$d\varphi = 0$

$ds^2 = (b^2 + a^2\cos^2\theta)\, d\theta^2$

integral elíptica completa

$$2\int_0^{\pi} d\theta \sqrt{b^2 + a^2\cos^2\theta} = 2\sqrt{a^2+b^2}\; E\left(\theta, \frac{a^2}{a^2+b^2}\right)\Big|_{\theta=0}^{\theta=\pi}$$

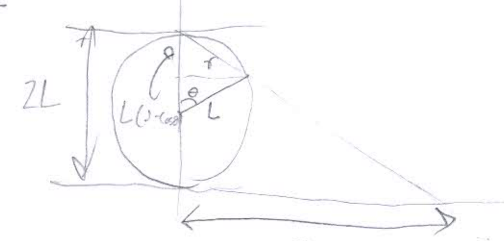
↳ 2 arcos iguais

$$a = b \rightarrow 2\sqrt{2}\, a\, E\left(\theta, \frac{1}{2}\right)\Big|_0^{\pi} = \underline{2\sqrt{2}\, a\, E\left(\tfrac{1}{2}\right)}$$

13) $ds^2 = \dfrac{dr^2}{1 - \frac{r^2}{L^2}} + r^2 d\varphi^2$

$L\sin\theta = dr$

$L\cos\theta = d\varphi$



similaridade triangular

$$\begin{cases} \dfrac{L(1-\cos\theta)}{r} = \dfrac{2L}{\rho} \\[2mm] \sin\theta = \dfrac{r}{L} \end{cases} \Rightarrow \boxed{r = \frac{\rho}{1 + \frac{\rho^2}{4L^2}}}$$

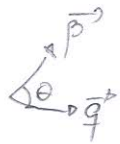
$$ds^2 = \frac{ds^2}{1 - r^2/L^2} + r^2 d\varphi^2 = \frac{1}{(1+\rho^2/4^2)^2}\left(d\rho^2 + \rho^2 d\varphi^2\right)$$

14) $g_{\mu\nu} = \Omega^2 \delta_{\mu\nu}$

$\downarrow$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = \Omega^2 dx^2$$

$$ds^2_{flat} = \delta_{\mu\nu}\, dx^\mu dx^\nu = dx^2$$

Ângulos $\vec{p}, \vec{q}$ 

6/4/2016

Superfícies de Minkowski

$$F(x^\mu) = 0$$

→ Tipo - espaço : a dist. infinitesimal entre 2 pontos da superfície é do tipo - espaço

$\begin{cases} \text{tangentes} \to \text{tipo espaço} \\ \text{normal} \to \text{tipo tempo} \end{cases}$

→ Nulas (ou tipo-Luz) : geradas por vetores tipo-luz

→ $\begin{cases} \text{normal é também de tipo Luz} \\ \text{e faz parte da superfície} \end{cases}$

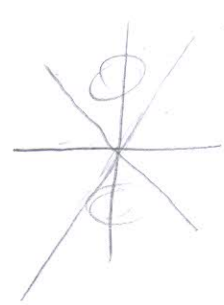
Exemplo : Minkowski em coord. esféricas

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

Cone de Luz:

$$\left.\begin{matrix} t = r \\ dt = dr \\ d\theta = 0 \\ d\varphi = 0 \end{matrix}\right\} ds^2 = 0$$



trajetória:

$$l^\mu = (1, 1, 0, 0)$$

$l^\mu l^\nu g_{\mu\nu} = 0$ → vetor tangente ao cone de Luz

os outros 2 vetores tangentes são:

$$h^\mu = \left(0, 0, \frac{1}{r}, 0\right) \quad \text{e} \quad k^\mu = \left(0, 0, 0, \frac{1}{r\sin\theta}\right)$$

$h^\mu h^\nu g_{\mu\nu} = 1 = k^\mu k^\nu g_{\mu\nu} \to$ tipo espaço $\oplus$ ortogonais a ℓ^+

$h^\mu \ell^\nu g_{\mu\nu} = 0 = k^\mu \ell^\nu g_{\mu\nu}$