23/4/2016

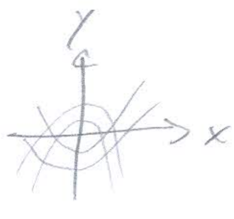
Correção da P1

1) $(x,y) \to (\mu,\nu)$

$$\begin{cases} x = \mu\nu \\ y = \frac{1}{2}(\mu^2-\nu^2) \end{cases}$$

$$ds^2 = dx^2 + dy^2 = (\mu d\nu + \nu d\mu)^2 + (\mu d\mu - \nu d\nu)^2$$

$$= \underbrace{(\mu^2+\nu^2)}_{\Omega}(d\mu^2 + d\nu^2)$$

b) 

$\mu = cte$

$x = \mu\nu \to \nu = \frac{x}{\mu}$

$$\to y = \frac{1}{2}\mu^2 - \frac{1}{2}\nu^2 = \frac{1}{2}\mu^2 - \frac{1}{2}\frac{x^2}{\mu^2}$$

$$= \underbrace{\underbrace{const}_{>0} - \underbrace{const}_{>0} \cdot x^2}_{<0}$$

c) i) conf. plana

ii) não tem um termo $d\mu d\nu$

d) $x^2 + y^2 = r^2$

$$\mu^2 c^2 + \frac{1}{4}(\mu^2 - \nu^2)^2 = r^2$$

$$\frac{1}{4}(\mu^2+\nu^2)^2 = r^2$$

$$\mu^2 + \nu^2 = 2r$$

novo raio $= \sqrt{2r}$

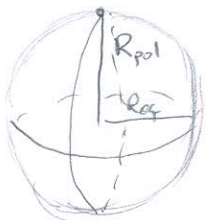
e) $$C = \oint ds = \oint (\mu^2+\nu^2)^{1/2}\sqrt{d\mu^2 + d\nu^2}$$

$$= \sqrt{2r}\oint \sqrt{d\mu^2 + d\nu^2} =$$

$$= \sqrt{2r}\oint d\mu \sqrt{1 + \left(\frac{d\nu}{d\mu}\right)^2}$$

$$= \sqrt{2r}\int_{-\sqrt{2r}}^{\sqrt{2r}} d\mu \sqrt{1 + \frac{\mu^2}{2r-\mu^2}} = \pi\sqrt{2r}$$

2) Equador



$$C_{eq} = \int_0^{2\pi} R f(\pi/2)\, d\phi = 2\pi f(\pi/2)$$

$$= 2\pi R(1+\epsilon)$$

$$\| \quad 2\pi R_{eq}$$

$$\Longrightarrow \boxed{R_{eq} = R(1+\epsilon)}$$

$R = 6357$ Km

$$\varepsilon = \frac{6378}{6357} - 1 \simeq 0{,}003$$

Longitude

$\phi = 0$

$$C_{pol} = 2\int_0^{\pi} R\, d\theta = 2\pi R$$

$$\| \quad 2\pi R_{pol} \Longrightarrow \boxed{R_{pol} = R}$$

3) (r, ψ) plano $\quad (\theta, \phi)$ esfera

$$r = r(\theta) \qquad \psi = \phi$$

a) $$ds^2 = R^2 (d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$= R^2 \left(\left(\frac{d\theta}{dr}\right)^2 dr^2 + \sin^2\theta\, d\psi^2 \right)$$

$$\theta = \theta(r)$$

b) Comprimento na esfera $\underline{dr}$ ao longo de r $(\psi = cte)$

$$R\left(\frac{d\theta}{dr}\right) dr$$

Comprimento $\underline{d\psi}$ ao longo de ψ ($r = cte$)

$$R \sin\Theta \, d\psi$$

$$r \perp \psi \rightarrow \text{area} = \boxed{R^2 \frac{d\Theta}{dr} \sin\Theta \, dr \, d\psi}$$

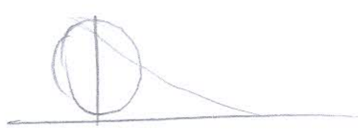
elemento de área na esfera

No plano dr $d\psi$

a area seria $\underbrace{dr}_{\psi = cte} \cdot \underbrace{r\,d\psi}_{r = cte}$

negativa; $-L$ $(L>0)$

$$R^2 \frac{d\Theta}{dr} \sin\Theta \, dr \, d\psi = k\, r \, dr \, d\psi$$



$r=0$ $\quad \Theta = 0 \rightarrow c = R^2$

$$R^2 \frac{d\Theta}{dr} \sin\Theta = -L r$$

$$R^2 \int d\Theta \, \sin\Theta = -L \int dr \, r$$

$$-R^2 \cos\Theta + a = -\frac{L r^2}{2} + b$$

$$-R^2 \cos\Theta + \underbrace{(a-b)}_{c = R^2} = -\frac{L r^2}{2}$$

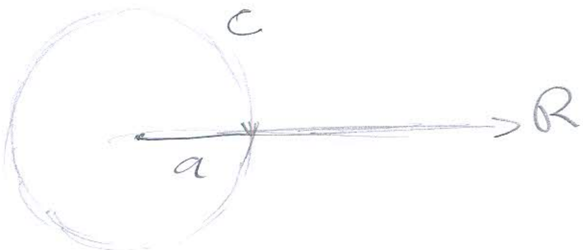
$$\frac{L r^2}{2} = R^2 (1 - \cos\Theta)$$

$$r = \sqrt{\frac{2R^2}{L}(1 - \cos\Theta)}$$

4) $ds^2 = \alpha^2 dr^2 + r^2 d\phi^2$

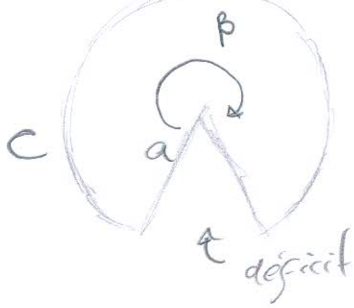
$\alpha > 1$

$$\left.\begin{aligned} R &\equiv \alpha r \\ \Phi &\equiv \phi/\alpha \end{aligned}\right\} \longrightarrow \underbrace{ds^2 = dR^2 + R^2 d\Phi^2}_{\text{plano exceto o ponto } R=0}$$



$C = \int a \, d\Phi = \dfrac{2\pi a}{\alpha}$

$R = \text{const} = a$



$\dfrac{c}{a} = \dfrac{2\pi}{\alpha} = 2\pi \sin \beta/2$

$$\boxed{\beta = 2 \sin^{-1}\left(\frac{1}{\alpha}\right)}$$

6) $$\left.\begin{aligned} -\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x} = 0 \\ L = \dot{x}^2 + x^2 \end{aligned}\right\} \rightarrow \boxed{\ddot{x} = x}$$

$x = \alpha \,\mathrm{sh}\, t + \beta \,\mathrm{ch}\, t$

$= \gamma e^t + \delta e^{-t}$

$$\left.\begin{aligned} x(0) &= 0 \\ x(T) &= 1 \end{aligned}\right\} \rightarrow \begin{aligned} \beta &= 0 \\ \alpha &= \frac{1}{\mathrm{sh}\, T} \end{aligned}$$

$$\boxed{x = \frac{\mathrm{sh}\, T}{\mathrm{sh}\, T}}$$

$$S = \dot{x}x\Big|_0^T + \int_0^T dt\, \underbrace{x(-\ddot{x}+x)}_{=0}$$

$$= \underline{\coth T}$$

mínimo $\quad \tilde{x}(t) = \frac{t}{T}$ respeita $\begin{cases} x(0)=0 \\ x(T)=1 \end{cases}$

$$S\left[\tilde{x} = \frac{t}{T}\right] = \frac{1}{T}\left(1+\frac{T^2}{3}\right) > \coth T$$

5) $ds^2 = \Omega^2(x)\, \delta_{\mu\nu}\, dx^\mu dx^\nu$

$$S = \sqrt{\Omega^2\, \delta_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu}\, d\lambda \qquad \dot{x}^\mu = \frac{dx^\mu}{d\lambda}$$

$$\frac{\delta L}{\delta x^\mu} = \frac{d}{d\lambda}\frac{\delta L}{\delta \dot{x}^\mu} \qquad \lambda = \tau \Rightarrow \sqrt{\ } = 1$$

$$\frac{d}{d\lambda}\frac{\cancel{2}\, \dot{x}_\mu \Omega^2}{\cancel{2}\sqrt{\ }} = \frac{\cancel{2}\,\Omega\, \partial_\mu \Omega\, \dot{x}^2}{\cancel{2}\sqrt{\ }}$$

$$\lambda = \tau \to \sqrt{\ } = 1$$

$$\frac{d}{d\tau}\left(\dot{x}_\mu \Omega^2\right) = \Omega\, \partial_\mu \Omega\, \dot{x}^2$$

$$\ddot{x}_\mu \Omega^2 + 2\Omega\, \dot{x}_\mu \underbrace{\frac{d}{d\tau}\Omega}_{= \partial_\nu \Omega\, \dot{x}^\nu}$$

$$\ddot{x}_\mu + \frac{2\,\partial_\nu \Omega}{\Omega}\dot{x}^\nu \dot{x}_\mu - \frac{\partial_\mu \Omega}{\Omega}\dot{x}^2 = 0$$

P2 $\boxed{L3 + L4 + L5}$ P.2 → 18/5

- Relatividade Restrita
- E/M (covariante)
- Eq. de Einstein

(6) $$S = -m \int \sqrt{-g_{\mu\nu}(x)\, dx^\mu dx^\nu}$$

com $\begin{cases} g_{00} = -\left(1 + \frac{2V}{m}\right) \\ g_{0i} = g_{i0} = 0 \\ g_{ij} = \delta_{ij} \end{cases}$

$$S = -m \sqrt{\left(1 + \frac{2V}{m}\right) dt^2 - d\vec{x}^2}$$

$V = -\frac{GMm}{r}$ pot. grav. de Newton

$$S = -m \int \sqrt{\left(1 - \frac{2GM}{r}\right) dt^2 - d\vec{x}^2}$$

a m cancela

se $d\vec{x}^2 = 0$

$$S = -m \int dt \sqrt{1 - \frac{2GM}{r}}$$

$$\simeq -m \int dt \left(1 - \frac{GM}{r}\right)$$

↳ $GM \ll r$

$$= -m \int d\tau$$

$$d\tau = dt \left(1 - \frac{GM}{r}\right)$$

$$\boxed{d\tau < dt}$$

relógios se atrasam em um potencial gravitacional

Com mais que uma partícula

$$S_G = -\sum_a m_a \int \sqrt{\left(1 + \frac{2V(x_a)}{m_a}\right) dt_a^2 - d\vec{x}_a^2}$$

$V(x_a) \propto m_a \rightarrow$ partículas com massas diferentes experimentam tempos iguais

$\rightarrow$ $\boxed{\text{massa gravitacional = massa inercial}}$

$$S = -\sum_a m_a \int \sqrt{-g_{\mu\nu}(x_a)\, dx_a^\mu dx_a^\nu}$$

independente das características da partícula "a"

$\rightarrow$ $\boxed{\text{Gravidade é universal}}$

comprimento de curvas em espaço-tempo curvo!

partículas em um campo gravitacional se propagam como em um espaço-tempo curvo!

Simetrias "escondidas"

(E) $V(x)\,dt \rightarrow A_\mu(x)\, dx^\mu$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ $\leftarrow$ field strength = tensor de campo

invariança de calibre (gauge)

Qual é o equivalente de $F_{\mu\nu}F^{\mu\nu}$ para a gravidade?

$-g_{\mu\nu}dx^{\mu}dx^{\nu}$ é invariante sob

TGC (Trans gerais de coord)

$x^{\mu} \Rightarrow \tilde{x}^{\mu}(x^{\nu})$