9/5/2016

Mathematica

<< EDCRGTCcode.m

coordinates = {t, r, θ, φ}

metricSh = DiagonalMatrix[{-A[r], B[r], r^2, r^2Sin[θ]^2}]

RGtensors[metricSh, coordinates]

Rdd → devolve tensor de Ricci

Rdd // MatrixForm

R → escalar de curvatura

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Resolução das listas

A guide to physics problems
part 1/2

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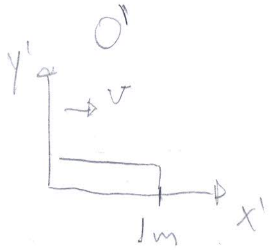
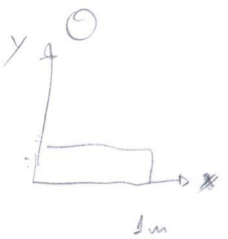
Lista 3:

Ex. 1 -




$L^2_i = (T, 0)$ $L^2_f = (T, 1)$

$L^1_i = (\gamma T, -\gamma v T)$

$L^1_f = (\gamma T - v T', \gamma(L - vT))$

$L^1_i = (T, 0)$ $L^1_f = (T, 1)$

$L^2_i = (\gamma T', \gamma(+vT'))$

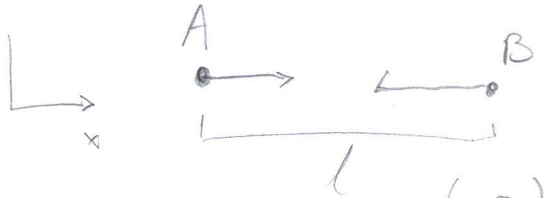
$L^2_f = (\gamma(\overset{=0}{T'} - vL), \gamma(L + vT'))$

$T' = -vL$

$\gamma(L - v^2 L) = L\sqrt{1-v^2} < L$

$T = 0$
$x = 0{,}8\,m$

Ex 2 -




$l = 4{,}2 \cdot 10^8\,m$

$v_A = 0{,}8\,c$

$v_B = -0{,}6\,c$

a) $\Delta t = \frac{L}{v} = \frac{4{,}2 \cdot 10^8}{1{,}2 \cdot 3 \cdot 10^8} = 1s$

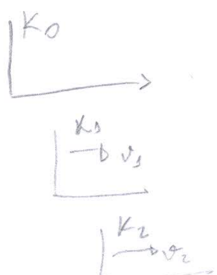
b) $v_B = -v_B$

$v'_B = \frac{-0{,}6c - 0{,}8c}{1 + \frac{v_A v_B}{c^2}} = -0{,}95c$

c) $t_A = \frac{t_T}{\gamma_A} = 1 \cdot \sqrt{1 - \frac{v_A^2}{c^2}} = 0{,}6s$

d) $t_B = \frac{t_T}{\gamma_B} = 0{,}8s$

Ex 3 -




P/ $K_1$ =

$v'_1 = \frac{v + v_1}{1 + \frac{v \cdot v_1}{c^2}}$

$\frac{v}{c} = \beta \left( \frac{v_1}{c} = \beta_1 \right| \frac{v_2}{c} = \beta_2$

P/ $K_2$ =

$v'_2 = \frac{v_2 + v'_1}{1 + \frac{v_2 v'_1}{c^2}}$

$= \beta_1 - \frac{\beta + \beta_2}{1 + \beta\beta_2} \Rightarrow \beta_0 = \frac{\beta + \beta_1}{1 + \beta\beta_1} = \frac{\beta_2 + \frac{2\beta}{1+\beta^2}}{1 + \frac{2\beta\beta_2}{1+\beta^2}}$

$\tanh \psi = \beta$

$$\tanh(\psi_1 + \psi_2) = \frac{\tanh \psi_1 + \tanh \psi_2}{1 + \tanh \psi_1 \tanh \psi_2}$$

$$\Rightarrow \beta_o = \tanh\left((n+1) \underbrace{\operatorname{arctanh} \beta_i}_{\psi}\right)$$

Ex 4 - Cons. mom          Centro de Massa $\rightarrow$ $v$

$$\underbrace{M(t+dt)}_{\substack{\parallel \\ M(t) + M'(t)\,dt}} dv - u\,|dM| = M(t+dt)\,dv + u\,dM = 0$$

$$\boxed{M\,dv = -u\,dM}$$ 1st order

Lab frame

$dv = dV$

$\frac{M}{M_0} = e^{-V/u}$

b) Rocket's frame

$M\,dV - \gamma_u\,dm \cdot u = 0$

$\gamma_u = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$          $M\,dv = -u\,dM$

$$\underbrace{V + dV}_{\text{Lab frame}} = \frac{V + dv}{1 + V\frac{dv}{c^2}} \longrightarrow dv = \gamma^2 dV \rightarrow \int \frac{dV}{1 - \frac{V^2}{c^2}} = \int -u\frac{dM}{M} \rightarrow$$

$$\rightarrow \boxed{\frac{M}{M_0} = \left(\frac{1-\beta}{1+\beta}\right)^{c/2u} \simeq e^{-V/u}}$$

Lista 4

Ex 1 - $(t, r, \theta, \phi) \to (T, \rho, \theta, \phi)$

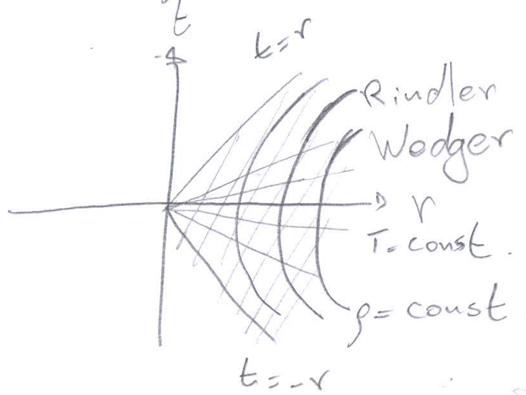
$$t = \rho \, \mathrm{sh}\, T \qquad dt = d\rho \, \mathrm{sh}\, T + \mathrm{ch}\, T \, \rho \, dT$$

$$r = \rho \, \mathrm{ch}\, T \qquad dr = d\rho \, \mathrm{ch}\, T + \mathrm{sh}\, T \, \rho \, dT$$

$$-dt^2 + dr^2 = d\rho^2 \underbrace{(-\mathrm{sh}^2 T + \mathrm{ch}^2 T)}_{=1} + dT^2 \overbrace{(-\rho^2 \mathrm{ch}^2 T + \rho^2 \mathrm{sh}^2 T)}^{-\rho^2}$$

$$+ d\rho \, dT \underbrace{(-2\rho \, \mathrm{sh}\, T \, \mathrm{ch}\, T + 2\rho \, \mathrm{ch}\, T \, \mathrm{sh}\, T)}_{=0}$$

$$ds^2 = -\rho^2 dT^2 + d\rho^2 + \rho^2 \mathrm{ch}^2 T \, (d\theta^2 + \sin^2\theta \, d\phi^2)$$




$(\theta, \varphi) = \text{const}$

Ex 5 - $F_{\mu\nu}F^{\mu\nu} = F_{00}\overset{=0}{F^{00}} + \underbrace{F_{0i}}_{-E_i}\underbrace{F^{0i}}_{+E_i} + \underbrace{F_{i0}F^{i0}}_{F_{0i}F^{0i}} + \underbrace{F_{ij}F^{ij}}_{-\epsilon_{ijk}B_k \epsilon^{ijx}B_x}$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & & \\ E_y & & 0 & \\ E_z & & & 0 \end{pmatrix}$$

$$\boxed{\mathcal{L} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2)} \qquad T^{00} \to H$$

b) $$\boxed{T^{\mu\nu} = F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4}\eta^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}}$$

$$T^{00} = F^{0}{}_{\lambda} F^{0\lambda} - \frac{1}{4}\eta^{00}(2\vec{E}^2 - 2\vec{B}^2) = \vec{E}^2 + \frac{1}{2}(\vec{B}^2 - \vec{E}^2) = \frac{1}{2}(B^2 + E^2)$$

$$\nabla\cdot\vec{B} = 0 \; (\rho_m)$$ (Dirac 1927)

$$\nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \; (\vec{j}_m)$$

Dualidade

E/M

para Maxwell

sem fontes

$$\nabla\cdot\vec{E} = \rho = 0 \; (\rho)$$

$$\nabla\times E - \frac{\partial B}{\partial t} = \vec{j} = 0 \; (\vec{j})$$

$$\boxed{E \to -\vec{B}}$$

$$\partial_\mu F^{\mu\nu} = 0$$

$$\partial_\mu \hat{F}^{\mu\nu} = 0$$

$$F_{\mu\nu} \leftrightarrow \hat{F}_{\mu\nu}$$

$$\rho \leftrightarrow \rho_m$$

$$\vec{j} \leftrightarrow \vec{j}_m$$