$$\nabla \cdot \vec{B} = 0 \; (\rho_m) \quad (\text{Dirac } 1927)$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \; (\vec{j}_m)$$

Dualidade E/M para Maxwell sem fontes

$$\nabla \cdot \vec{E} = \rho = 0 \; (\rho)$$

$$\nabla \times E - \frac{\partial B}{\partial t} = \vec{j} = 0 \; (\vec{j})$$

$$\boxed{E \to -\vec{B}}$$

$$\partial_\mu F^{\mu\nu} = 0$$

$$\partial_\mu \hat{F}^{\mu\nu} = 0 \qquad F_{\mu\nu} \leftrightarrow \hat{F}_{\mu\nu}$$

$$\rho \leftrightarrow \rho^m$$
$$j \leftrightarrow \vec{j}_m$$

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18/5/2016

## Resolução da lista 5

Ex 5. $T_{\mu\nu} = F_{\mu\nu} F_{\nu}^{\nu} - \frac{1}{4} g_{\mu\nu} F_{\sigma\tau} F^{\sigma\tau}$

$$D_\mu T^{\mu\nu} = 0$$

$$A^{\mu\nu} = g^{\mu\rho} g^{\nu\sigma} A_{\rho\sigma} \qquad B^\mu \quad C^\nu$$
$$= g^{\mu\rho} g^{\nu\sigma} (B_\rho C_\sigma) = g^{\mu\rho} B_\rho \, g^{\nu\sigma} C_\sigma$$

$$T^{\mu\nu} = F^{\mu}{}_{\sigma} F^{\nu\sigma} - \frac{1}{4} g^{\mu\nu} F_{\sigma\tau} F^{\sigma\tau}$$

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$$D_\mu F^{\mu\nu} = 0$$

$$D_\mu F_{\rho\sigma} + D_\rho F_{\sigma\mu} + D_\sigma F_{\mu\rho} = 0$$

$$D_\mu T^{\mu\nu} = D_\mu \Big( \underbrace{F^{\mu}{}_{\sigma} F^{\nu\sigma}}_{1} - \underbrace{\frac{1}{4} g^{\mu\nu} F_{\sigma\tau} F^{\sigma\tau}}_{2} \Big)$$

① $D_\mu(\cancel{F^\mu{}_\sigma}) F^{\nu\sigma} + \underbrace{F^{\mu\sigma} D_\mu F^\nu{}_\sigma}$

$F^{\mu\sigma} D_\mu F^\nu{}_\sigma$

$D_\mu g_{\sigma\lambda} F^{\nu\lambda}$

$F^{\mu\sigma} g_{\sigma\lambda} D_\mu F^{\nu\lambda}$

$$\Rightarrow F^{\mu\sigma} D_\mu F^\nu{}_\sigma = F^\mu{}_\sigma D_\mu g^{\nu\lambda} F_{\lambda\sigma} = F^{\mu\sigma} g^{\nu\lambda} D_\mu F_{\lambda\sigma}$$

$$= \frac{1}{2} F^{\mu\sigma} g^{\nu\lambda} D_\mu F_{\lambda\sigma} + \frac{1}{2} F^{\cancel{\mu\sigma}\,\sigma\mu} g^{\nu\lambda} D_{\cancel{\mu}\,\sigma} F_{\lambda\cancel{\sigma}\,\mu}$$

$$= \frac{1}{2} F^{\mu\sigma} g^{\nu\lambda} D_\mu F_{\lambda\sigma} + \frac{1}{2} F^{\mu\sigma} g^{\nu\lambda} D_\sigma F_{\cancel{\lambda\mu}\,\mu\lambda}$$

$$= \frac{1}{2} F^{\mu\sigma} g^{\nu\lambda} \left[ D_\mu F_{\lambda\sigma} + D_\sigma F_{\mu\lambda} \right]$$

$$= -D_\lambda F_{\sigma\mu}$$

$$① = -\frac{1}{2} F^{\mu\sigma} g^{\nu\lambda} D_\lambda F_{\sigma\mu}$$

No final

$$① + ② = \frac{1}{2} F^{\mu\beta} g^{\alpha\nu} (\text{Bianchi}) = 0$$

" " "

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$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \, (+\Lambda g_{\mu\nu}) = 0 \quad \# \quad \frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}_{maxwell}}{\delta g^{\mu\nu}} \propto T_{\mu\nu}$$

$$S = S_{EH} + S_{maxwell}$$

↳ grav. pura

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{\delta S_{EH}}{\delta g^{\mu\nu}} + \frac{\delta S_{maxwell}}{\delta g^{\mu\nu}} = 0$$

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_{maxwell}}{\delta g^{\mu\nu}}$$

$$S_{maxwell} = -\frac{1}{4}\int d^4x \sqrt{-g}\, F_{\mu\nu} F^{\mu\nu}$$

$$\boxed{\mathcal{L}_{em} = -\frac{1}{4}\sqrt{-g}\, F_{\mu\nu} F^{\mu\nu}} \qquad F_{\mu\nu}\, g^{\mu\mu'} g^{\nu\nu'} F_{\mu'\nu'}$$

$$\partial_\mu \to D_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\delta\Big(-\frac{1}{4}\overset{1}{\sqrt{-g}}\;\overset{2}{g^{\mu\alpha}}\;\overset{3}{g^{\nu\beta}}\;\underbrace{F_{\mu\nu} F_{\alpha\beta}}_{\equiv F^2}\Big)$$

$$\delta(\sqrt{-g}) = -\frac{\sqrt{-g}}{2}\, g_{\mu\nu}\, \delta g^{\mu\nu}$$

$$= +\frac{\sqrt{-g}}{2}\, g^{\mu\nu} g_{\mu\nu}$$

$$1 = -\frac{1}{4}\left(-\frac{\sqrt{-g}}{2}\right) g_{\lambda\tau}\, \delta g^{\lambda\tau} F^2$$

$$2 = -\frac{1}{4}\sqrt{-g}\left(\delta g^{\mu\alpha}\right) g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} \quad (\text{troca } \nu \text{ com } \alpha)$$

$$3 = -\frac{1}{4}\sqrt{-g}\, g^{\mu\alpha}\left(\delta g^{\nu\beta}\right) F_{\mu\nu} F_{\alpha\beta} \quad (\text{troca } \beta \text{ com } \mu)$$

$$\delta g^{\mu\nu}$$

$$1 + 2 + 3 = +\frac{1}{8}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu} F^2$$

$$- \frac{1}{4}\sqrt{-g}\left(g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} + g^{\alpha\beta} F_{\beta\nu} F_{\alpha\mu}\right)\delta g^{\mu\nu}$$

$$= \frac{1}{8}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu} F^2 - \frac{1}{2}\sqrt{-g}\, g^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu}\, \delta g^{\mu\nu}$$

$$= -\frac{\sqrt{-g}}{2}\,\delta g^{\mu\nu}\left(-\frac{1}{\cancel{8}4}\, g_{\mu\nu}F^2 + \frac{1}{2}\, F_{\mu}^{\ \alpha} F_{\alpha\nu}\right)$$

$$\mathcal{L}_{em} = -\frac{1}{4}\sqrt{-g}\, F_{\mu\nu}F^{\mu\nu}$$

$$\frac{\delta \mathcal{L}_{em}}{\delta g^{\mu\nu}} = \frac{\sqrt{-g}}{2}\Big(\underbrace{F_{\alpha\mu}F^{\alpha}{}_{\nu} - \frac{1}{4}\, g_{\mu\nu}F^2}_{=\,T_{\mu\nu}}\Big)\,\delta g^{\mu\nu}$$

$$= -\frac{\sqrt{-g}}{2}\, T_{\mu\nu}$$