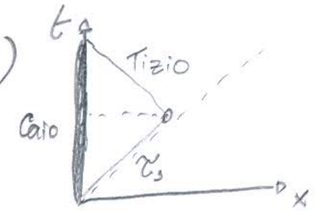
23/5/2016

Correção da prova

1) 

$\tau_1 = 7y$ (referencial próprio do Tizio)

$$\begin{cases} t_1 = \tau_1 \dfrac{1}{\sqrt{1+\left(\frac{24}{25}\right)^2}} = 25y \\ t_2 = 2t_1 = 50y \end{cases}$$

$$\left.\begin{array}{l} \text{Caio} \rightarrow 75y \\ \text{Tizio} \rightarrow \tau_1 + \tau_2 = 51y \end{array}\right\} 75-51 = \boxed{24y}$$

$$\tau_2 = 2t_1 \sqrt{1-\left(\frac{12}{25}\right)^2} = 44y$$

② Efeito Compton "inverso"

$p^\mu = (E, 0, 0, p) \leftarrow$ elétron

$k^\mu = (\omega, 0, 0, -\omega) \leftarrow$ fóton

$k^{\mu'} = (\omega', 0, \omega' \sin\theta, \omega' \cos\theta)$

$p^{\mu'}$

Conservação

$$p^\mu + k^\mu = p^{\mu'} + k^{\mu'}$$

$$p^{\mu'} = p^\mu + k^\mu - k^{\mu'}$$

$$(p^{\mu'})^2 = -m^2 = (p^\mu + k^\mu - k^{\mu'})^2 = -m^2 + 2p\cdot k - 2p\cdot k' - 2k\cdot k'$$

$$k\cdot k' = p\cdot k - p\cdot k' \rightarrow \omega' = \frac{\omega(E+p)}{E+\omega-(p-\omega)\cos\theta} \longrightarrow$$

$\rightarrow \omega'_{max} = \dfrac{\omega(E+p)}{E+\omega-p+\omega} \simeq \dfrac{E}{1+\frac{m^2}{4\omega E}}$ $\qquad p=\sqrt{E^2-m^2} \simeq E-\dfrac{m^2}{2E}$

$\hookrightarrow$ componente z de p^μ

$\omega \ll p \rightarrow p-\omega > 0 \quad \cos\Theta = 1 \rightarrow \boxed{\Theta = 0}$

back scattering

3) $\begin{cases} t = \left(\dfrac{c}{g}+\dfrac{x'}{c}\right) sh\left(\dfrac{gt'}{c}\right) \\ x = c\left(\dfrac{c}{g}+\dfrac{x'}{c}\right) ch\left(\dfrac{gt'}{c}\right) - \dfrac{c^2}{g} \end{cases}$ $\qquad \begin{matrix} dy' = dy \\ dz' = dz \end{matrix}$

$$dt = \frac{dx'}{c} sh\left(\frac{gt'}{c}\right) + \frac{g}{c}\left(\frac{c}{g}+\frac{x'}{c}\right) ch\left(\frac{gt'}{c}\right) dt'$$

$$dx = dx' ch\left(\frac{gt'}{c}\right) + g\left(\frac{c}{g}+\frac{x'}{c}\right) sh\left(\frac{gt'}{c}\right) dt'$$

$$-c^2dt^2 + dx^2 = \begin{cases} dt'^2\left(-g^2\left(\frac{c}{g}+\frac{x'}{c}\right)^2\right) \\ dx'^2\ (1) \\ dt'dx'\ (0) \end{cases}$$

a) $\boxed{-c^2dt^2 + dx^2 = -c^2\left(1+\dfrac{gx'}{c^2}\right)^2 dt'^2 + dx'^2 + dy'^2 + dz'^2}$

b) $gt' \ll c \qquad \begin{cases} t \simeq t' \\ x \simeq x' + \frac{1}{2}gt'^2 = x' + \frac{1}{2}gt^2 \end{cases}$

c) $(d\tau)_{x'=0} \overset{c=1}{=} dt'$ $\qquad (d\tau)_{x'=h} = \left(1+\dfrac{gh}{c^2}\right)(d\tau)_{x'=0}$

$(d\tau)_{x'=h} = dt'\left(1+\dfrac{gh}{c^2}\right)$

4) $d\sigma^2 = 0 \Rightarrow$ cone de luz

$$\begin{cases} d\sigma^2 = g_{\mu\nu}\, dx^\mu dx^\nu \\ d\sigma^2 = \Omega^2 g_{\mu\nu}\, dx^\mu dx^\nu \end{cases}$$

5) a) $A_\mu = (\phi, \vec{A})$

$$\frac{d\vec{p}}{dt} = -\frac{q}{c}\frac{\partial \vec{A}}{\partial t} - q\vec{\nabla}\phi - \frac{q}{c}\vec{v}\times(\nabla\times\vec{A}) -$$

$$= q\vec{E} + \frac{q}{c}\vec{v}\times\vec{B}$$

$$\begin{cases} \vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi \\ \vec{B} = \nabla\times\vec{A} \end{cases}$$

1D $E_x \neq 0$ $\frac{dp}{dt} = qE$

b) $v = \frac{dx}{dt} = \frac{dx/d\tau}{dt/d\tau}$ $\frac{dx}{dt} = c\,\mathrm{sh}\left(\frac{qE\tau}{mc}\right)$

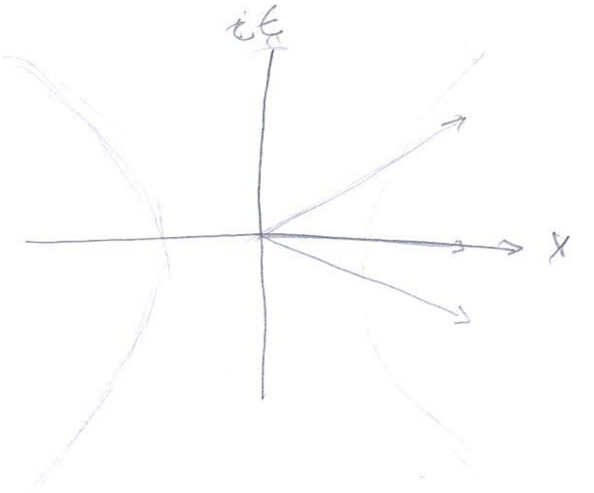
$$\frac{dt}{d\tau} = \mathrm{ch}\left(\frac{qE\tau}{mc}\right) \rightarrow \boxed{v = c\,\mathrm{th}\left(\frac{qE\tau}{mc}\right)}$$

$$1 - \mathrm{th}^2 = \frac{1}{\mathrm{ch}^2}$$

$$p = \frac{mc\,\mathrm{th}()}{\frac{1}{\mathrm{ch}()}} = mc\,\mathrm{sh}(\;)$$

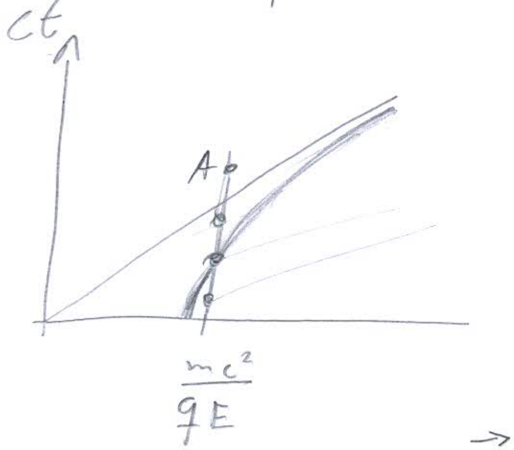
$$\frac{dp}{dt} = \frac{dp/d\tau}{dt/d\tau} = \frac{qE\,\mathrm{ch}()}{\mathrm{ch}()} = qE$$

$$w^\mu = \frac{1}{m}\frac{dp^\mu}{dt}$$



$$\boxed{w^\mu w_\mu = -\left(\frac{qE}{m}\right)^2}$$

$x^2 - c^2t^2 = \left(\frac{mc^2}{qE}\right)^2$ hyperbole



interseção do

$$\begin{cases} x = \frac{mc^2}{qE} & \text{(linha vertical)} \\ x = ct \end{cases}$$

$\rightarrow t = \frac{mc}{qE}$ $\quad ft = \frac{mcf}{qE}$ # de flashes antes do tempo A

(6) $R_{\mu\nu\rho\sigma}$ 1, 2

$$R_{1212} = R_{2121} = -R_{1221} = \dots$$

$$R_{\tau\rho\mu\nu} = f(R)\left(g_{\tau\mu} g_{\rho\nu} - g_{\tau\nu} g_{\rho\mu}\right)$$

$$R_{\rho\nu} = g^{\tau\mu} R_{\tau\rho\mu\nu} = f(R)\left(2 g_{\rho\nu} - \underbrace{\delta^\mu_\nu g_{\rho\mu}}_{g_{\rho\nu}}\right)$$

$$R = g^{\rho\nu} R_{\rho\nu} = f(R)(2\cdot 2 - 2) = 2f(R)$$

$$\boxed{f(R) = \frac{R}{2}}$$

$$g^{\rho\nu}\left[R_{\tau\rho\mu\nu} = \frac{R}{2}(g_{\tau\mu}g_{\rho\nu} - g_{\tau\nu}g_{\rho\mu}\right]$$

$$R_{\tau\mu} = g_{\tau\mu}\frac{R}{2}$$

$$\boxed{R_{\tau\mu} - g_{\tau\mu}\frac{R}{2} = 0}$$

$$\boxed{\int dx^2 \sqrt{-g}\, R}$$