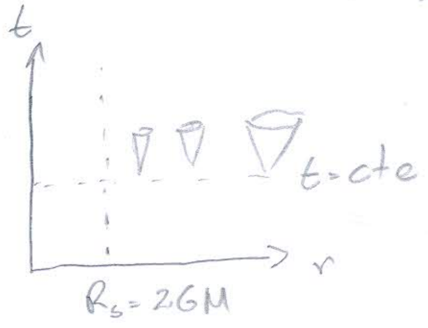
25/5/2016

Relatividade

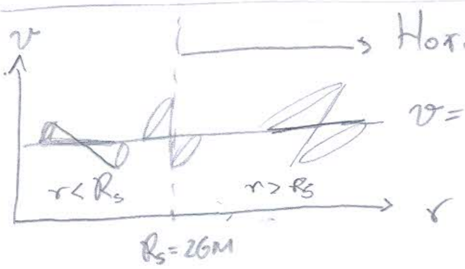
Schwarzschild

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2 d\Omega_2^2$$



E-F

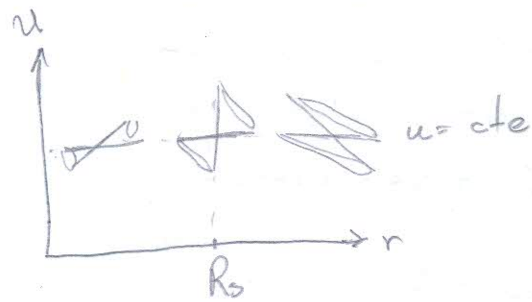
$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dv^2 + 2v\,dv\,dr + r^2 d\Omega_2^2$$



Extensões? sim.

Por exemplo, poderíamos ter usado (u,r)

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)du^2 - 2du\,dr + r^2 d\Omega_2^2$$



(u,v)

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)du\,dv + r^2 d\Omega_2^2$$

$r = r(u,v)$

Com

$$\frac{1}{2}(v-u) = r + 2GM \ln\left(\frac{r}{2GM} - 1\right)$$

degenerecência

$$r = R_s : \begin{cases} v = -\infty \\ \text{ou} \\ u = +\infty \end{cases}$$

$$\begin{cases} v' = e^{v/4GM} = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{\frac{r+t}{4GM}} \\ u' = -e^{-u/4GM} = -\left(\frac{r}{2GM} - 1\right)^{1/2} e^{\frac{r-t}{4GM}} \end{cases}$$

$$ds^2 = -\frac{16 G^3 M^3}{r} e^{-r/2GM} \left(2 du' dv'\right) + r^2 d\Omega_2^2 \Rightarrow$$

$$r = R_s \ \underline{\text{ok}} \ \Big\} \Rightarrow ds^2 = \frac{32 G^3 M^3}{r} e^{-r/2GM} \left(-d\tau + dR^2\right) + r^2 d\Omega_2^2$$

KRUSKL-SZEKERES

$$\begin{cases} T = \frac{1}{2}(v' + u') = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{r/4GM} \, sh\left(\frac{t}{4GM}\right) \\ R = \frac{1}{2}(v' - u') = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{r/4GM} \, ch\left(\frac{t}{4GM}\right) \end{cases}$$

$$r = r(R, T)$$
$$\boxed{\text{com } T^2 - R^2 = \left(1 - \frac{r}{2GM}\right) e^{r/2GM}}$$

u', v' : coordenadas nulas
θ, φ : " tipo-espaço
} → seria melhor ter uma coordenada de tipo tempo e 3 tipo-espaço

<u>Propriedades</u>

1) Curvas nulas radiais são sempre 45°

$$\left.\begin{matrix} \theta = cte \\ \phi = cte \end{matrix}\right| \to d\Omega_2^2 = 0$$

$$ds^2\Big|_{\substack{\theta = cte \\ \phi = cte}} = 0 \qquad T = \pm R + cte$$

2) $r = R_s$ é infinitamente longe

$$T = \pm R$$

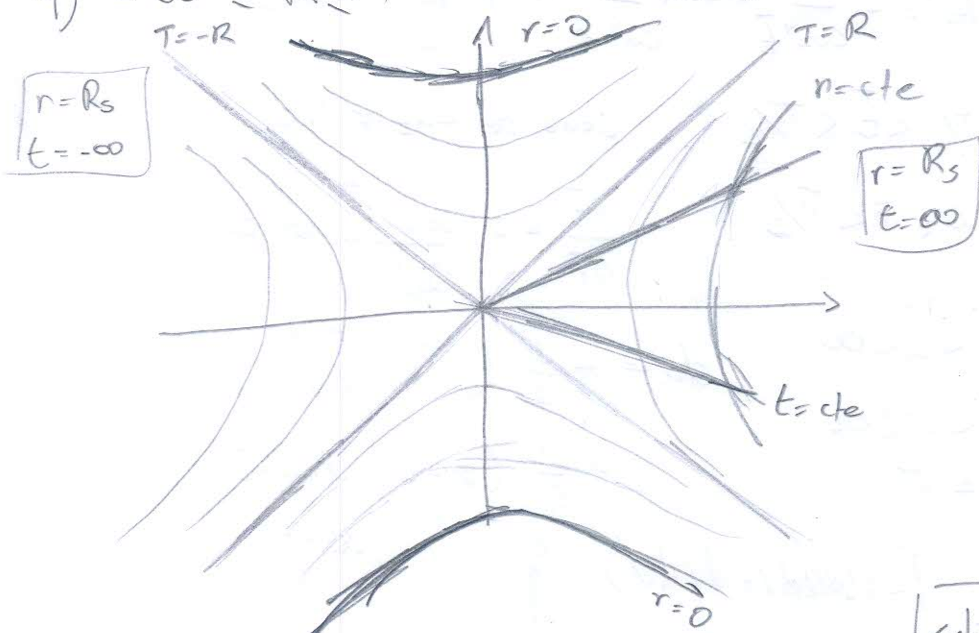
3) $r = cte \rightarrow T^2 - R^2 = cte \rightarrow$ hiperboles

$t = cte \rightarrow \dfrac{T}{R} = th \dfrac{t}{4GM} \rightarrow$ linhas retas passando pela origem com "slope" $th\, t/4GM$

$t \rightarrow \pm\infty \qquad \dfrac{T}{R} = \pm 1$

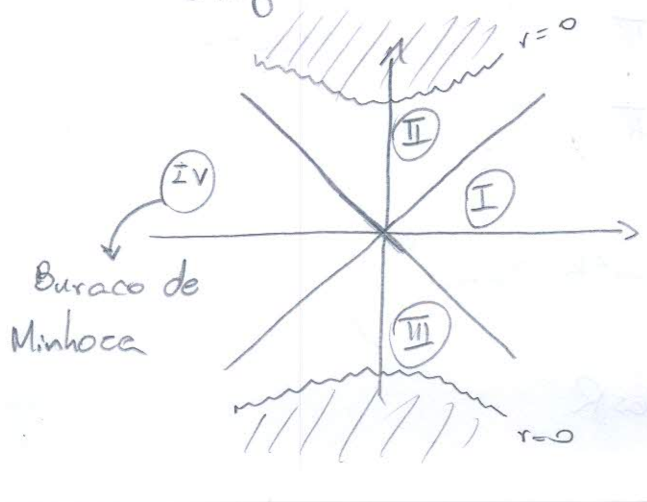
$t = \pm\infty$ é a mesma superfície de $r = R_s$

4) $-\infty \leq R \leq +\infty \qquad T^2 < R^2 + 1$



$\boxed{\text{schwarzschild (I)} \quad r > 2GM = R_s}$

Diagrama de Kruskal



(II): Buraco Negro

(III) Buraco Branco (Não-físico)

(IV) Cópia de (I), mas sem contato causal

Diagramas conformes ou de Penrose
ou de Carter - Penrose

*) Cone de luz a 45°

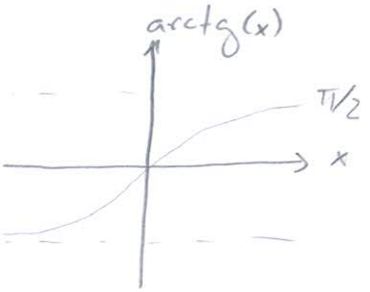
*) "infinitos à distância finita"

1) Minkowski

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2$$

Cone de luz : 45°

$$\begin{cases} -\infty < t < \infty \\ 0 \le r < \infty \end{cases}$$



$$\begin{cases} \bar{t} = \operatorname{arctg} t \\ \bar{r} = \operatorname{arctg} r \end{cases}$$

$$ds^2 = -\frac{1}{\cos^4 \bar{t}} d\bar{t}^2 + \frac{1}{\cos^4 \bar{r}} d\bar{r}^2 + \operatorname{tg}^2 \bar{r}\, d\Omega_2^2$$

$$\begin{cases} -\pi/2 < \bar{t} < \pi/2 \\ 0 \le \bar{r} < \pi/2 \end{cases}$$

Cone de luz $\neq$ 45°

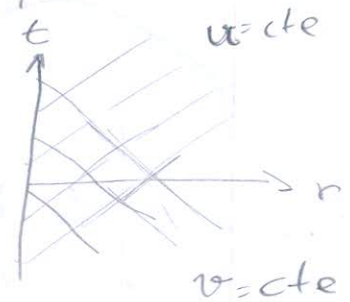
$$\frac{d\bar{t}}{d\bar{r}} = \pm \frac{\cos^4 \bar{t}}{\cos^4 \bar{r}} \neq \pm 1$$

$$\longrightarrow \quad u = t - r \qquad v = t + r \qquad \begin{cases} -\infty < u < \infty \\ -\infty < v < \infty \end{cases} \quad u \le v$$

$$ds^2 = -\frac{1}{2}\left(du\,dv + dv\,du\right) + \frac{1}{4}(v-u)^2 d\Omega_2^2$$

$$\begin{cases} U = \operatorname{arctg} u \\ V = \operatorname{arctg} v \end{cases}$$

$$ds^2 = \frac{1}{4\cos^2 U \cos^2 V}\left[-2(dU\,dV + dV\,dU) + \sin^2(V-U)\, d\Omega_2^2\right]$$



$$\begin{cases} -\frac{\pi}{2} < U < \pi/2 \\ -\pi/2 < V < \pi/2 \\ U \le V \end{cases}$$

$$\downarrow$$

$$\begin{cases} T = V + U \\ R = V - U \end{cases} \qquad \begin{cases} 0 \le R \le \pi \\ |T| + R < \pi \end{cases}$$

$$ds^2 = w^{-2}(T,R)\left(-dT^2 + dR^2 + \sin^2 R\, d\Omega_2^2\right)$$

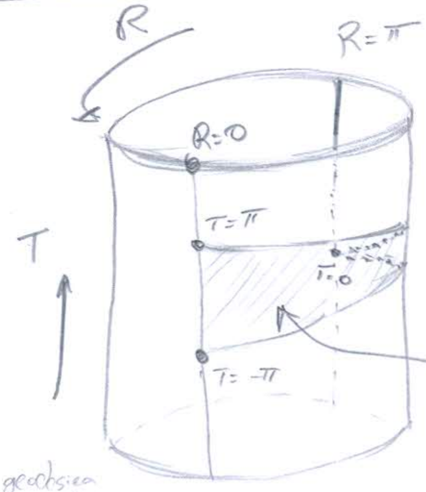
$$w = \cos T + \cos R$$

$$d\tilde{s}^2 = -dT^2 + dR^2 + \sin^2 R \, d\Omega_2^2 = \omega^2 ds^2$$

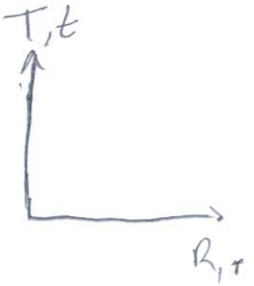
Métrica não-física

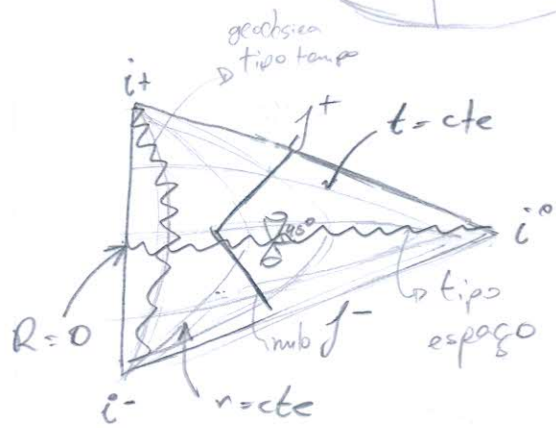
$\boxed{\mathbb{R} \times S^3}$

Universo estático de Einstein



conformemente Minkowski





i^+ = inf. fut. de tipo tempo $(T=\pi,\ R=0)$

i^0 = inf. espacial $(T=0,\ R=\pi)$

i^- = infinito passado de tipo tempo $(T=-\pi,\ R=0)$

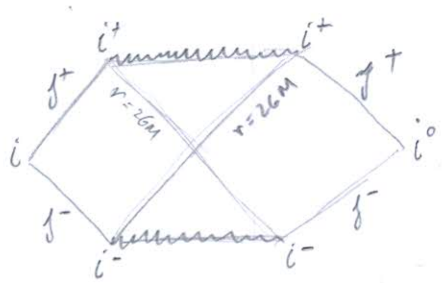
$\mathcal{J}^+$ = infinito futuro de tipo nulo $\begin{pmatrix} T=\pi-R \\ 0<R<\pi \end{pmatrix}$

$\mathcal{J}^-$ = infinito passado nulo $\begin{pmatrix} T=-\pi+R \\ 0<R<\pi \end{pmatrix}$

Schwarzschild

$$ds^2 = -\frac{16GM^3}{r} e^{-r/4GM}\left(2\,dv'du'\right) + r^2 d\Omega_2^2$$

$$\begin{cases} v'' = \operatorname{arctg}\left(\frac{v'}{\sqrt{2GM}}\right) \\ u'' = \operatorname{arctg}\left(\frac{u'}{\sqrt{2GM}}\right) \end{cases} \qquad \begin{cases} -\frac{\pi}{2} < v'' < \frac{\pi}{2} \\ -\pi/2 < u'' < \pi/2 \\ -\frac{\pi}{2} < v'' + u'' < \pi/2 \end{cases}$$



$\boxed{i^\pm \neq r=0}$