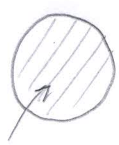
6/6/2016

Equação TOV



$T_{\mu\nu} \neq 0$

$R_{\mu\nu} = 0$

$$T_{\mu\nu} = \begin{pmatrix} \rho(x) & 0 & 0 & 0 \\ 0 & p(x) & 0 & 0 \\ 0 & 0 & p(x) & 0 \\ 0 & 0 & 0 & p(x) \end{pmatrix}$$

KERR

$R_{\mu\nu} = 0$

$$\begin{cases} t \to -t \\ \phi \to -\phi \end{cases}$$

$dt\,d\phi$

M, J

4D "No-hair theorem"

$\downarrow$

"3 cabelos" M J Q

1916 1918

Reissner - Norstrom

$Q \neq 0$

$T_{\mu\nu} \neq 0$

Estático + $T_{\mu\nu} \neq 0$

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}}$$

Acoplamento Gravidade + Matéria

$$S = S_{EH} + S_{MAT}$$

↳ Maxwell, partículas, ...

$$\delta S_{MAT} = \frac{1}{2} \int d^4x \sqrt{-g}\; T^{\mu\nu}(x)\; \delta g_{\mu\nu}(x)$$

$$\boxed{T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S_{mat}}{\delta g_{\mu\nu}}}$$

$$\delta S = \delta S_{EM} + \delta S_{mat}$$

$$= \int d^4x \sqrt{-g} \Big\{ \underbrace{-\frac{1}{16\pi G}\left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R\right) + \frac{1}{2} T^{\mu\nu}}_{=0} \Big\} \delta g^{\mu\nu}$$

HW: Partículas

$$S_{part} \doteq -\sum_a m_a \int d\tau_a \sqrt{-g_{\mu\nu}(x_a) \frac{dx^\mu_a}{d\tau_a} \frac{dx^\nu_a}{d\tau_a}} \to T^{\mu\nu} = \ldots$$

Objetivo: Resolver usando $T^{\mu\nu}$ do e/m

$$g^{\mu\nu}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) = 8\pi G\, g^{\mu\nu} T_{\mu\nu} \quad = \text{Traço} = T$$

$$R - \frac{1}{2} 4R = 8\pi G T$$

$$\rightarrow \boxed{R = -8\pi G T \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)}$$

$T_{maxwell} = 0$

Traço

$$T_{\mu\nu} = F_{\mu\lambda} F^\lambda{}_\nu - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$$

$$g^{\mu\nu} T_{\mu\nu} = T = F_{\mu\nu}F^{\mu\nu} - \frac{4}{4} F_{\rho\sigma}F^{\rho\sigma}$$

$= 0$

$$\longrightarrow T = 0$$

$$\rightarrow \begin{cases} R_{\mu\nu} = 8\pi G\, T_{\mu\nu} \\ \boxed{D_\mu F^{\mu\nu} = 0} = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, F^{\mu\nu} \right) \end{cases}$$

sim. esf. + estático

$$ds^2 = -A(r)\, dt^2 + B(r)\, dr^2 + r^2 d\Omega_2^2$$

Ansatz para o campo e/m

campo elétrico

$$F_{0i} \longrightarrow \boxed{F_{0r}} \qquad \boxed{E(r)}$$

$$\downarrow$$

$$\begin{cases} F_{0r} \\ F_{0\theta} \\ F_{0\phi} \end{cases}$$

$$E = F_{0r} = -F_{r0}$$

$$\begin{cases} F^{0r} = g^{00} g^{rr} F_{0r} = \frac{E(r)}{A(r)B(r)} \longrightarrow \\ g = \det g_{\mu\nu} = -A(r)B(r)\, r^4 \sin^2\theta \end{cases}$$

$$\partial_r \left(\sqrt{g}\, F^{r0} \right)$$

$$\longrightarrow \partial_r \left(\frac{r^2 E}{\sqrt{AB}} \right) = 0$$

solução

$$E = \frac{\sqrt{AB}}{r^2} Q$$

Q ↳ cte

↳ HW: Bianchi $\underline{Or}$

HW

$$\begin{cases} T_{00} = \ldots = \frac{E^2}{2B} \\ T_{rr} = \frac{-E^2}{2A} \\ T_{\theta\theta} = \frac{r^2 E^2}{2AB} \qquad T_{\phi\phi} \propto T_{\theta\theta} \end{cases}$$

$$\boxed{R_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

Mathematica

$A \cdot B = cte = 1$

↳ para ter um espaço-tempo assint. plano

$$\boxed{A = \frac{1}{B}}$$

$$T_{\theta\theta} = \frac{r^2 E^2}{2AB} = \frac{r^2}{2AB}\frac{AB}{r^4}Q^2 = \frac{Q^2}{2r^2}$$

absorve

$$A(r) = \frac{1}{B(r)} = 1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}$$

$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2 d_{\Omega}\Omega_2^2$$

$$\boxed{A = \frac{1}{B} = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}}$$

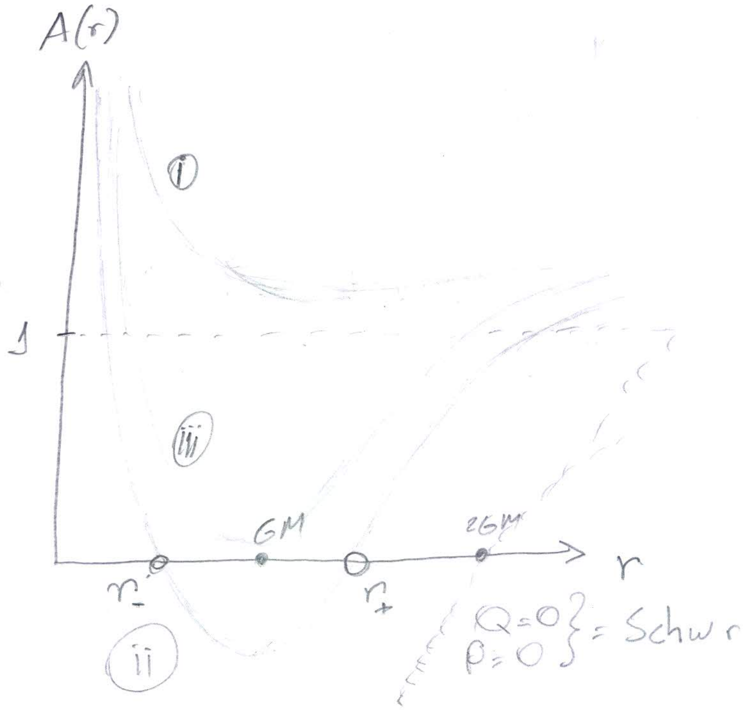
$$\left(\text{Campo magnético} \rightarrow F_{\theta\phi} = -F_{\phi\theta} \rightarrow \frac{GQ^2}{r^2} \rightarrow \frac{G(Q^2+P^2)}{r^2} \right)$$

Singularidades

$$\boxed{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \xrightarrow[r\to 0]{} \infty}$$

$r = 0$ sing. de curvatura

$$\boxed{A(r) = 0}$$



$r_\pm = GM \pm \sqrt{G^2M^2 - GR^2}$

(i) $\boxed{GM^2 < Q^2}$

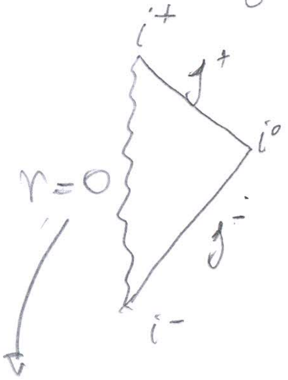
Trans - Extremal ou Naked

Métrica é sempre regular até $r=0$

$\begin{cases} t = \text{sempre time-like} \\ r = \text{" space-like} \end{cases}$

Não tem horizonte → singularidade nua

$r \to \infty$: espaço tempo plano



"Censura cósmica"

sing. de tipo tempo, nua, na origem

(ii) $GM^2 > Q^2$

Sub - Extremal

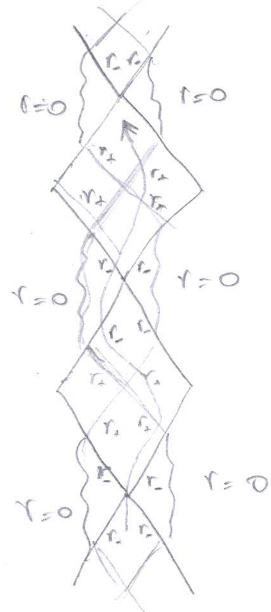
$r = r_\pm$ são nulos → horizonte de eventos

$r=0$ singularidades de tipo - tempo

(r_+) r vai de tipo espaço para tipo tempo

$r < r_+$ o observador vai para r menor necessariamente

(v) r volta a ser do tipo espaço!

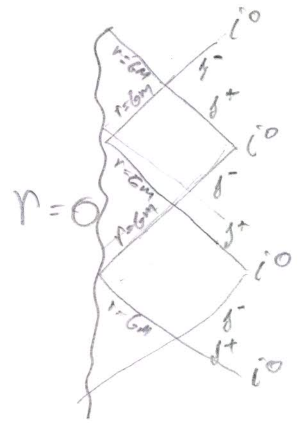


(iii) $GM^2 = Q^2$ Extremal

$r_+ = r_- = GM$

r = Nunca de tipo tempo

vira nula em $r = GM$



$$A(r) = 1 - \frac{2GM}{r} + \frac{G^2M^2}{r^2} = \left(1 - \frac{GM}{r}\right)^2$$

$\rho = r - GM$

$d\rho = dr$

$$\to ds^2 = -\left(\frac{\rho}{\rho + GM}\right)^2 dt^2 + \left(\frac{\rho + GM}{\rho}\right)^2 d\rho^2 + \frac{(\rho + GM)^2}{\rho^2}\rho^2 d\Omega_2^2$$

$$= -H(\rho)^{-2} dt^2 + H^2(\rho)\left(d\rho^2 + \rho^2 d\Omega_2^2\right) \quad \boxed{H(\rho) = 1 + \frac{GM}{\rho}}$$

Buraco Negro ou RN extremal

(ex lista 7)

Coord. cartesianas

$$ds^2 = -H(\vec{x})^{-2}\,dt^2 + H(\vec{x})^2\,d\vec{x}^{\,2}$$

$$\boxed{H = 1 + \frac{GM}{|\vec{x}|}}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{0r} = \cancel{\partial_0 A_r} - \partial_r A_0$$

↳ Nada dep. de t

$$E = F_{r0} = \frac{Q}{r^2} = \partial_r A_0$$

$$\boxed{E = \partial_r A_0}$$

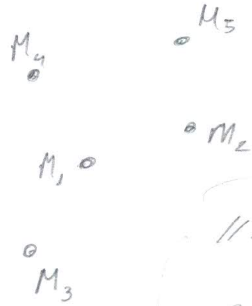
$$A_0 = -\frac{Q}{\rho + GM} \overset{\text{extremal}}{=} \frac{-\sqrt{G}\,M}{\rho + GM}$$

<u>HW</u>: Maxwell

Você vai descobrir que as eq. de Maxwell são resolvidas também por uma coleção de Buracos Negros

$$H = 1 + \sum_{a=1}^{N} \frac{GM_a}{|\vec{x} - \vec{x}_a|}$$

também é uma solução



Buracos negros extremos com massa M_a localizados em $\vec{x}_a$

multi-centered BH

"BPS"

BPSness

supersimetria

$M = Q$

Schw — Geodésicas

Buracos Negros (Penrose)

TOV

RN

grav. + Maxwell

Matéria da Prova