Energia Escura

$$P = -\rho \rightarrow \frac{d\rho}{dt} = 0 \rightarrow \boxed{\rho_{EE} \propto a^0}$$

13/6/2016

Lista 7 -

Ex 1 - $ds^2 = A(r)dt^2 + B(r)(dr^2 + r^2 d\Omega_2^2)$

$= dx^2 + dy^2 + dz^2$

coord. isotrópicas

fator conforme

$\lambda(\rho) \quad \rho = \rho(r)$

$$\begin{cases} ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + [\lambda(\rho)]^2(d\rho^2 + \rho^2 d\Omega_2^2) \\ \quad = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2 d\Omega_2^2 \end{cases}$$

$$\begin{cases} [\lambda(\rho)]^2 \rho^2 = r^2 \\ \frac{dr^2}{1 - \frac{2GM}{r}} = [\lambda(\rho)]^2 d\rho^2 \end{cases}$$

$$\pm \frac{d\rho}{\rho} = \frac{dr}{\sqrt{r^2 - 2GMr}}$$

$r \rightarrow \infty$

$\Rightarrow \rho \rightarrow \infty \Rightarrow +$

$$\Rightarrow \boxed{r = \rho\left(1 + \frac{GM}{2\rho}\right)^2}$$

$$\Rightarrow [\lambda(\rho)]^2 = \left(1 + \frac{2GM}{2\rho}\right)^4 \Rightarrow ds^2 = -\frac{\left(1 - \frac{GM}{2\rho}\right)^2}{\left(1 + \frac{GM}{2\rho}\right)^2} dt^2 + \left(1 + \frac{GM}{2\rho}\right)^4 (d\rho^2 + \rho^2 d\Omega_2^2)$$

Ex 2 - Zee VII. 1

obs: $S = -m\int ds \to \tilde{S} = \frac{1}{2}\int d\tau\,(e^{-1}\dot{x}^2 - m^2 e)$ e = campo auxiliar

a) $ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d_2\Omega_2^2$

$\tau \to \sqrt{\ } = 1$

massa m

$$L = \left(A(r)\left(\frac{dt}{d\tau}\right)^2 - B(r)\left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\theta}{d\tau}\right)^2 - r^2\sin^2\theta\left(\frac{d\phi}{d\tau}\right)^2\right)^{1/2}$$

$$= \sqrt{-\frac{ds^2}{d\tau}}$$

$\cancel{t}$ $\cancel{\phi}$ $\to$ 2 ctes do movimento

$$\begin{cases} \frac{d}{d\tau}\left(A(r)\frac{dt}{d\tau}\right) = \frac{dL}{dt} = 0 \quad (*) \\ \frac{d}{dt}\left(B\frac{dr}{d\tau}\right) + \frac{1}{2}A'\left(\frac{dt}{d\tau}\right)^2 - \frac{1}{2}B'\left(\frac{dr}{d\tau}\right)^2 - r\left(\frac{d\theta}{dr}\right)^2 - r\sin^2\theta\left(\frac{d\phi}{d\tau}\right)^2 = 0 \\ \frac{d}{d\tau}\left(r^2\frac{d\theta}{d\tau}\right) - r^2\sin\theta\cos\theta\left(\frac{d\phi}{d\tau}\right)^2 = 0 \\ \frac{d}{d\tau}\left(r^2\sin^2\theta\,\frac{d\phi}{d\tau}\right) = \frac{dL}{d\phi} = 0 \quad (**) \end{cases}$$

$$\begin{cases} A(r)\,\frac{dt}{d\tau} = cte \equiv \varepsilon \quad (*) \\ r^2\sin^2\theta\,\frac{d\phi}{d\tau} = cte \equiv \ell \quad (**) \end{cases}$$

Ex3 - $(G=1)$

$\circ\, 10M$

$r=2M$

Radial $\Rightarrow l=0$

$$u^t = \left(1-\frac{2M}{r}\right)^{-1/2}\Bigg|_{r=10M} = \frac{\sqrt{5}}{2}$$

$$\mathcal{E} = \left(1-\frac{2M}{r}\right) u^t \Bigg|_{r=10M}$$

$$\tau = \int_0^{10M} dr \left(\frac{2M}{r} - \frac{1}{5}\right)^{-1/2}$$

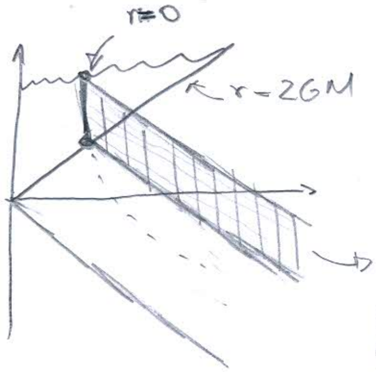
$\hookrightarrow r = 10M\xi$

$$u\cdot u = -1 \Rightarrow \left(\frac{dr}{d\tau}\right)^2 = \mathcal{E}^2 - \left(1-\frac{2M}{r}\right) = \frac{2M}{r} - \frac{1}{5} =$$

4-vel.

$$= 10\sqrt{3}\,M \int d\xi \left(\frac{1}{\xi} - 1\right)^{-1/2} = \underline{5\sqrt{3}\,\pi M}$$

Ex4 - KS



→ região que o observador pode receber informação

Ex5 - $\dfrac{dr}{dv} = \dfrac{1}{2}\left(1 + \dfrac{2GM}{r}\right) > 0 \Rightarrow$ raios de luz são sempre outgoing

$|M| > 0$

$v = cte \rightarrow$ ingoing

$$1 - \frac{2M}{r} \xrightarrow{M<0} 1 + \frac{2|M|}{r}$$

A cada ponto tem um raio de luz outgoing

 $\Rightarrow$ Não é um BN

Ex 10 - $ds^2 = -\left(1-\frac{M}{r}\right)^2 dt^2 + \left(1-\frac{M}{r}\right)^{-2} dr^2 + r^2 d\Omega_2^2$

$(G=1)$

a) $t = v - F(r) \Rightarrow dt = dv - F'dr \qquad F' = \frac{dF}{dr}$

$$ds^2 = -\left(1-\frac{M}{r}\right)^2 dv^2 + 2F'\left(1-\frac{M}{r}\right)^2 dv\,dr$$
$$+ \underbrace{\left(\left(1-\frac{M}{r}\right)^{-2} - (F')^2\left(1-\frac{M}{r}\right)^2\right)}_{=0 \;\Rightarrow\; \boxed{F' = \left(1-\frac{M}{r}\right)^{-2}}} dr^2 + r^2 d\Omega_2^2$$

$$= -\left(1-\frac{M}{r}\right)^2 dv^2 + 2\,dv\,dr + r^2 d\Omega_2^2$$

Não-singular em $\underline{r=M}$ (métrica fica não-degenerada $\det \neq 0$)

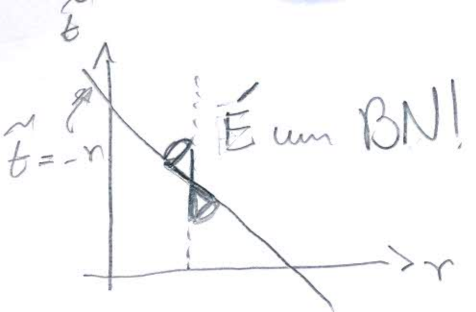
$$\begin{pmatrix} -\left(1-\frac{M}{r}\right)^2 & 1 & 0 & 0 \\ & 0 & 0 & 0 \\ 1 & & r^2 & 0 \\ 0 & 0 & & r^2\sin^2\theta \\ 0 & 0 & & \end{pmatrix}$$

b) Integrar $F' = \left(1-\frac{M}{r}\right)^{-2}$

$$\Rightarrow t = v - (r-M) - 2M\log\left|\frac{r}{M}-1\right| + \frac{M^2}{r-M}$$

$$t = r = 0 : v = 0$$

<u>Cone de luz</u> $\begin{cases} v = cte \\ \frac{dv}{dr} = \frac{2}{\left(1-\frac{M}{r}\right)^2} \end{cases}$ ou para $\tilde{t} = v - r$ $\begin{cases} \frac{d\tilde{t}}{dv} = -1 \\ \frac{d\tilde{t}}{dv} = -1 + \frac{2}{\left(1-\frac{M}{r}\right)^2} \end{cases}$

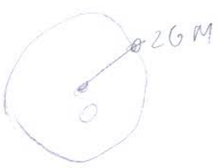


É um BN!

$$\frac{d\tilde{t}}{dr} = -1$$

Ex 11 – (G = 1)

$$\tau = -\int_{2GM}^{0} \frac{dr}{\left(\frac{dr}{d\tau}\right)}$$



$$= -\int_{2M}^{0} dr \left(\varepsilon^2 - \left(1 + \frac{\ell^2}{r^2}\right)\left(1 - \frac{2M}{r}\right)\right)^{-1/2}$$

verificar

$$= \int_{0}^{2M} dr \left(\underbrace{\varepsilon^2 + \left(1 + \frac{\ell^2}{r^2}\right)\left(\frac{2M}{r} - 1\right)}_{\text{o menor possível}}\right)^{-1/2}$$

$$\boxed{\begin{aligned} \varepsilon &= 0 \\ \ell &= 0 \end{aligned}}$$

$$= \int_{0}^{2M} dr \left(\frac{2M}{r} - 1\right)^{-1/2} = 2M \int_{0}^{1} d\xi \frac{\sqrt{\xi}}{\sqrt{1-\xi}} = \pi M$$

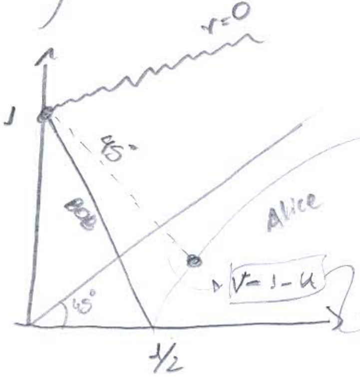
Ex 12 – Trajetória de tipo-tempo $u \cdot u = -1$
(G = 1)

$$\underbrace{\frac{32M^3 e^{-r/2M}}{r}\left(-\left(\frac{dv}{d\tau}\right)^2 + \left(\frac{du}{d\tau}\right)^2\right)}_{\text{tem que ser negativo}} + \underbrace{r^2\left(\frac{d\theta}{d\tau}\right)^2}_{\geq 0} + \underbrace{r^2 \sin^2\theta \left(\frac{d\phi}{d\tau}\right)}_{\geq 0} = -1$$

$$\left(\frac{dv}{d\tau}\right)^2 > \left(\frac{du}{d\tau}\right)^2 \Rightarrow \boxed{\left(\frac{dv}{du}\right)^2 > 1}$$

13)

a)



R tal que $\sqrt{\frac{R}{2M}-1}\, e^{R/4M} = \frac{1}{2}$

$\left(\frac{R}{2M}-1\right)^{1/2} e^{R/4M} \operatorname{sh}\left(\frac{t}{4M}\right) = 1 - \left(\frac{R}{2M}-1\right)^{1/2} e^{R/4M} \operatorname{ch}\left(\frac{t}{4M}\right)$

b) Bob $\rightarrow$ linha de mundo tem pendência $= +2$

$\Rightarrow$ dentro do cone de luz

$\Rightarrow$ traj. de tipo tempo

c) t máximo $=$ t no qual a linha $45°$ desde

$u=0,\ v=1$ interseca a curva $r=R$

$$\boxed{t = 4M \log 2}$$