Relatividade - 04/04/16 Exercícios

4) D $[J(mn), J(pq)]$ $D \xleftrightarrow{=} \frac{D(D-1)}{2}$ em $D=3$
Pag51

Caso $D=3$ $[J_i, J_j] = i\hbar \epsilon_{ijk} J_k$

7) $SO(3)$
Pag61
T^{ijk} Antisimétrico $i,j,k = 1,2,3$

$T^{123} = -T^{213} = T^{231} \ldots$

$T^{123} = (T^{123})' = R^{1i} R^{2j} R^{3k} T^{ijk}$

$\ldots = T^{123}$ Antisimétrico

18) $ds^2 = e^{2u}(du^2 + dv^2)$
Pag79

$= g_{\mu\nu} dx^\mu dx^\nu \Rightarrow g_{\mu\nu} = e^{2u} \delta_{\mu\nu}$

$= dx^2 + dy^2$

$= dr^2 + r^2 d\theta^2$

$(u, v) \longrightarrow (x, y)$

$u = \log r \longrightarrow e^u = r$

$du = \frac{1}{r} dr$

$dr = e^u du$

$(u, v) \Rightarrow \begin{pmatrix} u = \log r \\ v = \theta \end{pmatrix}$ $x = e^u \cos v$
$y = e^u \sin v$

Logo o espaço é plano

2) Pag 79

$$ds^2 = \left(1 - \frac{y^2}{3}\right)dx^2 + \left(1 - \frac{x^2}{3}\right)dy^2 + \frac{2}{3}xy\,dx\,dy + \dots$$

$$\begin{pmatrix} 1-\frac{y^2}{3} & \frac{xy}{3} \\ \frac{xy}{3} & 1-\frac{x^2}{y} \end{pmatrix} \qquad \begin{cases} x = \theta \sin\varphi \\ y = \theta \sin\theta \end{cases} \qquad g_{\mu\nu} = \frac{\partial x'^\rho}{\partial x^\mu}\frac{\partial x'^\sigma}{\partial x^\nu} g_{\rho\sigma}$$

$$g_{\theta\theta} = \frac{\partial x'^\rho}{\partial\theta}\frac{\partial x'^\sigma}{\partial\theta} g_{\rho\sigma} = \left(\frac{\partial x}{\partial\theta}\right)^2 g_{xx} + 2\frac{\partial x}{\partial\theta}\frac{\partial y}{\partial\theta} g_{xy} + \left(\frac{\partial y}{\partial\theta}\right)^2 g_{yy}$$

$$= \cos^2\varphi\left(1 - \frac{\theta^2\sin^2\varphi}{3}\right) + \frac{2}{3}\cos\varphi\sin\varphi + \sin^2\varphi\left(1 - \frac{\theta^2\cos^2\varphi}{3}\right)$$

$$= 1$$

7) Pag 94

$$ds^2 = g_{xx}\,dx^2 \qquad d\tilde{x} = \sqrt{g_{xx}}\,dx$$

$$= d\tilde{x}^2 \qquad \tilde{x} = \int^x dx\,\sqrt{g_{xx}}$$

$$\boxed{\begin{array}{l} D = 1 \longrightarrow 0 \\ \text{Riemann}(D) = \dfrac{D^2(D^2-1)}{12} \end{array}}$$

8) Pag 94

$$ds^2 = dx^2 + dy^2$$

$$z = y^p \longrightarrow z^{1/p} = y$$

$dy = dz$ [g_{zz} vai ter problemas em $z = 0$]

11) Pag 94

$$D = 2 \qquad ds^2 = \Omega\,(dx^2 + dy^2)$$

$$\text{Riemann}(2) = \frac{4(4-1)}{12} = 1$$

12) Pag 94 $\quad ds^2 = \dfrac{dx^2 + dy^2}{y^2} \qquad \Omega = \dfrac{1}{y^2}$

$$ds^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$$
$$= \Omega\,(dx^2 + dy^2)$$
(projeção estereográfico)

Lista 2

$D \cdot \dfrac{D(D+1)}{2}$

1) $x^{\mu} = x'^{\mu} + L^{\mu}_{\nu\lambda} x'^{\nu} x'^{\lambda} + \cdots$

$$g_{\mu\nu}(x) = \delta_{\mu\nu} + A_{\mu\nu\lambda} x^{\lambda} + \cdots$$

Escolher $L^{\mu}_{\nu\lambda}$ tais que $A_{\mu\nu\lambda} = 0$

$D \cdot \dfrac{D(D+1)}{2}$

2) geodésica na esfera : p 127

3) $\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})$

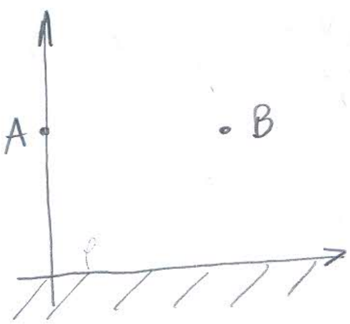
$g_{\mu\nu} = a_\mu \delta_{\mu\nu}$ (não tem soma sobre μ)

$$= \begin{pmatrix} a_1 & & 0 \\ & a_2 & \\ 0 & & \ddots \; a_D \end{pmatrix}$$

$$= \frac{1}{2} \underbrace{g^{\rho\sigma}}_{(a_\rho)^{-1}\delta^{\rho\sigma}} (\partial_\mu a_\nu \delta_{\nu\sigma} + \partial_\nu a_\mu \delta_{\mu\sigma} - \partial_\sigma a_\mu \delta_{\mu\nu})$$

$$= \frac{1}{2} (a_\rho)^{-1} \delta^{\rho\sigma} (\partial_\mu a_\nu \delta_{\nu\sigma} + \partial_\nu a_\mu \delta_{\mu\sigma} - \partial_\sigma a_\mu \delta_{\mu\nu})$$

4)



$$d = \int_A^B ds = \int_A^B \sqrt{\frac{dx^2+dy^2}{y^2}}$$

$$\underset{\lambda = y}{=} \int_a^b dy \sqrt{\frac{1+\left(\frac{dx}{dy}\right)^2}{y^2}}$$

$$\frac{d}{dy}\left(\frac{\delta L}{\delta \frac{dx}{dy}}\right) = \overset{=0}{\frac{\delta L}{\delta x}}$$

$$\frac{d}{dy}\underbrace{\left(\frac{dx/dy}{y\sqrt{1+(dx/dy)^2}}\right)}_{=\frac{1}{b}} = 0$$

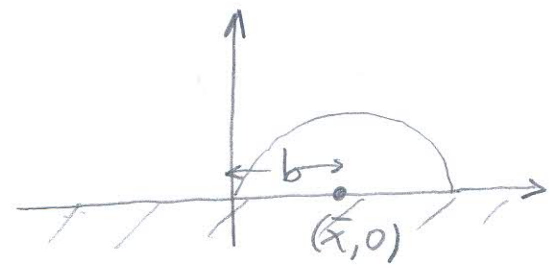
$$\frac{dx}{dy} = \frac{y}{b}\sqrt{1+\left(\frac{dx}{dy}\right)^2}$$

solução

$$x - \bar{x} = \pm\sqrt{b^2 - y^2}$$

Const. de integração

Semicírculos



⊛ As geodésicas são segmentos dos círculos.

5) eq. 30 p132

6) $S[x] = \int_{t_i=0}^{t_f} dt\, \frac{1}{2} m(\dot{x} - w^2 x^2)$

$\bar{x}$: Solução clássica $\quad \Delta S[\delta x] \equiv S[\bar{x}+\delta x] - S[\bar{x}]$

(usando as eqs do mov.) ↳ começa com termo quadrático $(\delta x)^2$ (ou $\left(\frac{d}{dt}\delta x\right)^2$) e não tem termo linear $\sim \delta x$ pois $\bar{x}$ é uma solução das eqs do Mov.

$$\Delta S[\delta x] = \int_0^{t_f} dt\, \frac{1}{2} m \left(\left(\frac{dx}{dt}\right)^2 - \omega^2 \delta x^2 \right)$$

$$\boxed{\delta_m x = \sin \omega_m t} \qquad \omega_m = \frac{\pi m}{t_f} \qquad \begin{cases} \delta_m x(t=0) = 0 \\ \delta_m x(t=t_f) = \sin(\pi m) = 0 \end{cases}$$

$$\Delta S[\delta_m x] = \int_0^{t_f} dt\, \frac{1}{2} m \left(\omega_m^2 \cos^2 \omega_m t - \omega^2 \sin^2 \omega_m t \right)$$

$$= \dots = \frac{1}{4} m (\omega_m^2 - \omega^2) t_f \qquad \left(\int_0^{t_f} \cos 2\omega_m t = 0 \right)$$

$$\delta x = \sum_{m=1}^{\infty} c_m \delta_m x = c_1 \delta_1 x + c_2 \delta_2 x + \cdots$$

$$\Delta S[\delta x] = \int_0^{t_f} dt\, \frac{1}{2} m \left(\left(\frac{d}{dt}(c_1 \delta_1 x + c_2 \delta_2 x + \dots) \right)^2 - \omega^2 (c_1 \delta_1 x + \dots)^2 \right)$$

Termos misturados não contribuem $\left(\frac{d}{dt} b_i \delta_i x + \frac{d}{dt} b_j \delta_j x \right)^2$

$$\int_0^{t_f} dt\, b_i b_j \left(\frac{d}{dt} \sin \omega_i t \right) \left(\frac{d}{dt} \sin \omega_j t \right)$$

$$= b_i b_j \omega_i \omega_j \int_0^{t_f} dt \cos \omega_i t \cos \omega_j t = 0 \quad \text{se } i \neq j$$

(HW)

$$\Rightarrow \Delta S[\delta x] = \int_0^{t} dt\, \frac{1}{2} m \left(\sum_i \left(\frac{d}{dt} c_i \delta_i x \right)^2 - \omega^2 \sum_i (c_i \delta_i x)^2 \right)$$

$$= \sum_{i=1}^{\infty} \Delta S[c_i \delta_i x]$$

$$\Rightarrow \boxed{\Delta S\left[\sum_i c_i \delta_i x\right] = \sum_i \Delta S[c_i \delta_i x] = \sum_i c_i^2 \Delta S[\delta_i x]}$$

$$\Delta S\left[\sum_n c_n \delta_n x\right] = \sum_n c^2 \Delta S[\delta_n x]$$

$$= \sum_n c_n^2 \frac{1}{4} m t_f (\omega_n^2 - \omega^2)$$

$$= \frac{1}{4} m t_f \sum_n c_n^2 (\omega_n^2 - \omega^2)$$

$\Delta S > 0 \quad \forall c_n \quad$ se $\omega_n > \omega \quad \forall n$

$\omega_n \sim n \qquad \omega_1 > \omega \longrightarrow \frac{\pi}{t_f} > \omega \longrightarrow \boxed{t_f < \frac{\pi}{\omega}}$

$\omega_n = \frac{\pi n}{t_f}$

Posso também escolher t_f tal que a solução é um ponto de sela.

P_1: 20/04 Listas 1 + 2

P_2 ~ metade de maio

P_3 ~ metade de Junho

Sub → 3ª semana de Junho