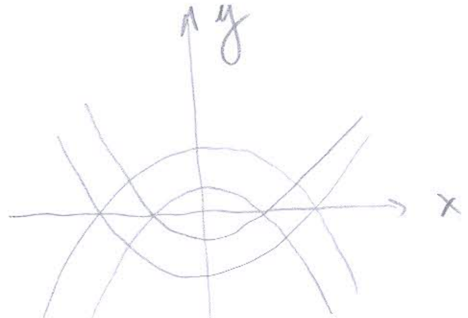
Relatividade 25/04 - Correção da prova + Notas

1) $(x, y) \rightarrow (\mu, v)$ $\begin{cases} x = \mu v \\ y = \frac{1}{2}(\mu^2 - v^2) \end{cases}$

a) $ds^2 = dx^2 + dy^2 = (\mu dv + v d\mu)^2 + (\mu d\mu - v dv)^2$

$= \underbrace{(\mu^2 + v^2)}_{\Omega^2}(d\mu^2 + dv^2)$

b) 

$\mu = \text{const.}$

$\rightarrow y = \frac{1}{2}\mu^2 - \frac{1}{2}v^2$

$= \frac{1}{2}\mu^2 - \frac{1}{2}\frac{1}{\mu^2}x^2$

$= \underbrace{\text{const}}_{>0} - \underbrace{\underbrace{\text{const}}_{>0} x^2}_{<0}$

c) i) conf. plana

ii) não tem um termo $d\mu dv$

d) $x^2 + y^2 = r^2 \rightarrow \mu^2 v^2 + \frac{1}{4}(\mu^2 - v^2)^2 = r^2$

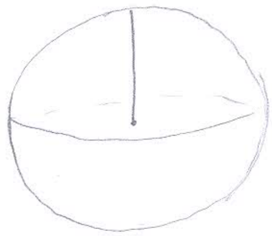
$\frac{1}{4}(\mu^2 + v^2) = r^2 \rightarrow \mu^2 + v^2 = 2r$ novo raio $= \sqrt{2r}$

e) $C = \oint ds = \oint (\mu^2 + v^2)^{1/2}\sqrt{d\mu^2 + dv^2}$

$= \sqrt{2r}\oint\sqrt{d\mu^2 + dv^2} = \sqrt{2r}\oint d\mu\sqrt{1 + \left(\frac{dv}{d\mu}\right)^2}$

$= \sqrt{2r}\int_{-\sqrt{2r}}^{\sqrt{2r}} d\mu\sqrt{1 + \frac{\mu^2}{2r - \mu^2}} = \sqrt{2r}\,\pi\sqrt{2r} = 2\pi r$

2) Equador: $\theta = \frac{\pi}{2}$



$$C_{eq} = \int_0^{2\pi} R\, f(\pi/2)\, d\phi = 2\pi R\, f(\pi/2)$$
$$= 2\pi R (1+\epsilon)$$

$$C_{eq} = 2\pi R_{eq}$$

$$\Rightarrow R_{eq} = R(1+\epsilon)$$

Longetudinal: $\phi = 0$

$$C_{pol} = 2\int_0^{\pi} R\, d\theta = 2\pi R \Rightarrow \boxed{R_{pol} = R}$$

$$\boxed{\epsilon = \frac{6378}{6357} - 1}$$

3) (r, ψ) Plano $\quad (\theta, \phi)$ esfera $\quad r = r(\theta)$, $\psi = \phi$

a)

$$ds^2 = R^2 (d\theta^2 + \sin^2\theta\, d\phi^2)$$
$$= R^2 \left(\left(\frac{d\theta}{dr}\right)^2 dr^2 + \sin^2\theta(r)\, d\psi^2 \right)$$

b) Comprimento na esfera $\underline{dr}$ ao longo de r (ψ = const.)

$$R\left(\frac{d\theta}{dr}\right) dr$$

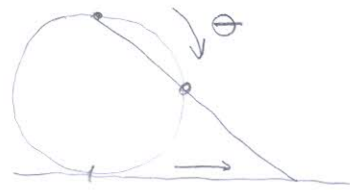
comp. $\underline{d\psi}$ ao longo de ψ (r = const.)

Elemento de área na esfera

$R \sin\theta\, d\psi$ $\qquad r \perp \psi \rightarrow \boxed{\text{área} = R^2 \frac{d\theta}{dr} \sin\theta\, dr\, d\psi}$

No plano (dr) (dψ)

a área seria $\underbrace{dr}_{\psi = \text{const.}} \cdot \underbrace{r\,d\psi}_{r = \text{const.}}$



$$R^2 \frac{d\theta}{dr} \sin\theta \, dr \, d\theta = K r \, dr \, d\psi$$

→ Negativa $(-L)$

$r \uparrow \quad \theta \downarrow$

$$R^2 \frac{d\theta}{dr} \sin\theta = -Lr \qquad R^2 \int d\theta \, \sin\theta = -L \int dr \, r$$

$$-R^2 \cos\theta + a = -L \frac{r^2}{2} + b$$

$$-R^2 \cos\theta + \underbrace{(a-b)}_{c} = -L\frac{r^2}{2} \xrightarrow{r=0,\,\theta=0} c = R^2$$

$$\frac{Lr^2}{2} = R^2(1-\cos\theta) \rightarrow r = \sqrt{\frac{2R^2}{L}(1-\cos\theta)}$$

4) $ds^2 = \alpha^2 dr^2 + r^2 d\phi^2 \qquad \alpha > 1$

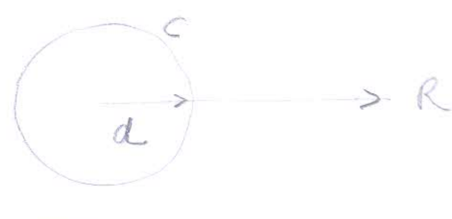
$$\left.\begin{aligned} R &\equiv \alpha r \\ \Phi &\equiv \phi/\alpha \end{aligned}\right\} \longrightarrow ds^2 = dR^2 + R^2 d\Phi^2$$

Plano exceto o ponto $R=0$

$$C = \int_0^{2\pi/\alpha} a \, d\Phi = \frac{2\pi a}{\alpha}$$

$R = \text{const} = a$

$$\frac{C}{a} = \frac{2\pi}{\alpha}$$



$$\frac{c}{a} = 2\pi \sin \beta/2 = \frac{2\pi}{\alpha} \Rightarrow \beta = 2 \sin^{-1}\left(\frac{1}{\alpha}\right)$$

6) $$-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x} = 0 \longrightarrow \ddot{x} = x$$

$$L = \dot{x}^2 + x^2$$

$x = \alpha \, \mathrm{sh}\, t + \beta \, \mathrm{ch}\, t$ mais geral

ou $= \gamma e^{t} + \delta e^{-t}$

$$\begin{matrix} x(0) = 0 \\ x(T) = 1 \end{matrix} \longrightarrow \begin{matrix} \beta = 0 \\ \alpha = \frac{1}{\mathrm{sh}T} \end{matrix} \Rightarrow x = \frac{\mathrm{sh}\, t}{\mathrm{sh}\, T}$$

$$S\left[x = \frac{\mathrm{sh}t}{\mathrm{sh}T}\right] = \dot{x} x \Big|_0^T + \int_0^T dt \, x \underbrace{(-\ddot{x} + x)}_{=0}$$

$$= \coth T$$

<u>mínimo</u> $\tilde{x}(t) = \frac{t}{T}$ respeita $\begin{cases} x(0) = 0 \\ x(T) = 1 \end{cases}$ mas

$$S\left[\tilde{x} = \frac{t}{T}\right] = \frac{1}{T}\left(1 + \frac{T^2}{3}\right) > \coth T$$

5) $$ds^2 = \Omega^2 \delta_{\mu\nu} dx^\mu dx^\nu$$

$$S = \int \sqrt{\Omega^2 \delta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \, d\lambda \qquad \dot{x}^\mu = \frac{dx^\mu}{d\lambda}$$

$$\frac{\delta L}{\delta x^\mu} = \frac{d}{d\lambda} \frac{\delta L}{\delta \dot{x}^\mu} \qquad \lambda = \tau \Rightarrow \sqrt{} = 1$$

$$\frac{d}{d\lambda} \frac{2\dot{x}_\mu \Omega^2}{2\sqrt{\quad}} = \frac{2\Omega\,\partial_\mu \Omega}{2\sqrt{\quad}} \dot{x}^2$$

$$\lambda = \tau \rightarrow \sqrt{\quad} = 1$$

$$\frac{d}{d\tau}\left(\dot{x}_\mu \Omega^2\right) = \Omega\,\partial_\mu \Omega\, \dot{x}^2$$

$$\ddot{x}_\mu \Omega^2 + 2\Omega \dot{x}_\mu \underbrace{\frac{d}{d\tau}\Omega}_{\partial_\nu \Omega \dot{x}^\nu}$$

$$\boxed{\ddot{x}_\mu + 2\frac{\partial_\nu \Omega}{\Omega} \dot{x}^\nu \dot{x}_\mu - \frac{\partial_\mu \Omega}{\Omega} \dot{x}^2 = 0}$$

<u>P2</u> → • Relatividade Restrita

• E/M (covariante)

• Eq. de Einstein

Listas $\boxed{L3 + L4 + L5}$

P2 → 18/05

<u>Aula</u>.

(G) $S = -m \int \sqrt{-g_{\mu\nu}(x)\, dx^\mu dx^\nu}$

Com $\begin{cases} g_{00} = -\left(1 + \frac{2V}{m}\right) \\ g_{0i} = g_{i0} = 0 \\ g_{ij} = \delta_{ij} \end{cases}$

$$S = -m\sqrt{\left(1 + \frac{2V}{m}\right) dt^2 - d\vec{x}^2}$$

$V = -\frac{GMm}{r}$ pot. gravit. de Newton

$$S = -m \int \sqrt{\left(1 - \frac{2GM}{r}\right) dt^2 - d\vec{x}^2}$$

<u>m</u> <u>cancela</u>

$$S = -m \int dt \sqrt{1 - \frac{2GM}{r}}$$

$$\underset{GM \ll r}{\simeq} -m \int dt \left(1 - \frac{GM}{r}\right) = -m \int d\tau$$

$$d\tau = dt \left(1 - \frac{GM}{r}\right)$$

$\boxed{d\tau < dt}$ → relógios atrasam em um pot. gravitacional

om mais do que uma partícula

$$S_G = -\sum_a m_a \int \sqrt{\left(1 + \frac{2V(x_a)}{m_a}\right) dt_a^2 - d\vec{x}_a^{\,2}}$$

$V(x_a) \propto m_a$ → partículas com massas diferentes
xperimentam tempos iguais

→ $\boxed{\text{massa gravitacional} = \text{massa inercial}}$
Eötvos (experimentos)

$$= -\sum_a m_a \int \sqrt{-g_{\mu\nu}(x_a)\, dx_a^\mu\, dx_a^\nu}$$

↳ independente das características da partícula "a".

$\boxed{\text{ravidade é universal}}$

Comprimento de curvas em espaço-tempo curvo!

$\boxed{\text{Partículas em um campo gravitacional se propagam como em um espaço-tempo curvo}}$

Simetrias "escondidas"

(E) $V(x)\,dt \longrightarrow A_\mu(x)\,dx^\mu$

$$\downarrow$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

↳ field strength (tensor de campo)

invariância de calibre (gauge)

fixou a ação $\longrightarrow$ $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Qual é o equivalente de $F_{\mu\nu}F^{\mu\nu}$ para a Gravidade?

Transformações gerais de coord.

$g_{\mu\nu}\,dx^\mu dx^\nu$ é invariante sob TGC

$$x^\mu \longrightarrow \tilde{x}^\mu(x^\nu)$$

(R) escalar de curvatura