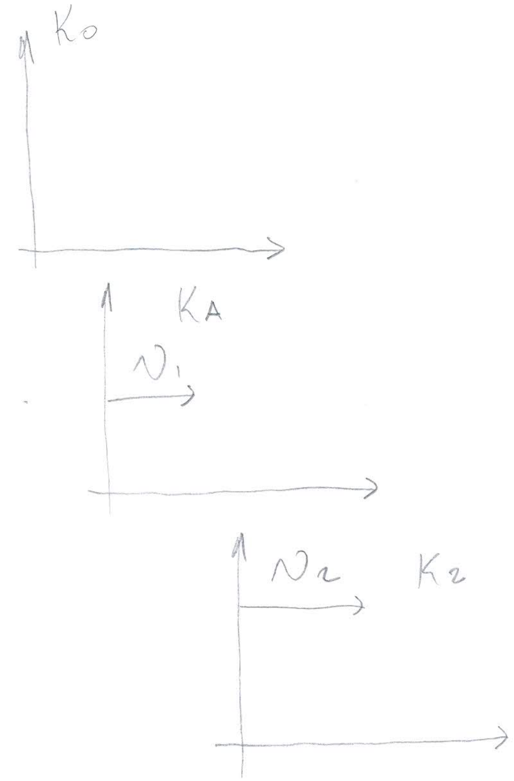
Relatividade - 9/05

→ A Guide to physics Problems | Sidney B. Cohn
Part 1/2 | B. Nadgorny

Q3.



P/K_1 :

$$N_1' = \frac{N + N_1}{1 + \frac{N N_1}{C^2}}$$

P/ K_2 :

$$N_2' = \frac{N_2 + N_1'}{1 + \frac{N_2 N_1'}{C^2}}$$

$\frac{N}{C} = \beta$ | $\frac{N_1}{C} = \beta_1$ | $\frac{N_2}{C} = \beta_2$

$$\Rightarrow \beta_1 - \frac{\beta + \beta_2}{1 + \beta\beta_2} = \frac{\beta + \beta_1}{1 + \beta\beta_1} = \frac{\beta_2 + 2\beta/(1+\beta^2)}{1 + 2\beta\beta_2/(1+\beta^2)}$$

→ Mudando a notação

b) $\tanh \Psi = \beta$

$$\tanh(\Psi_1 + \Psi_2) = \frac{\tanh \Psi_1 + \tanh \Psi_2}{1 + \tanh \Psi_1 \tanh \Psi_2}$$

$$\beta_0 = \tanh\left(\sum_c \Psi\right)$$

$$\beta_0 = \tanh((m+1) \tanh^{-1} \beta_i)$$

Ex 4 - Cons. momento (centro de Massa) → v

$$\underline{\underline{CM}}\quad \underbrace{M(t+dt)}\, dv - u\,|dM| = M(t+dt)\, dv + u\, dM = 0$$

$$\overset{\shortparallel}{M(t) + M'(t)\, dt}$$

$$\Rightarrow \boxed{M\, dv = -u\, dM} \; 1^a \text{ ordem}$$

Ref. Lab.

$$\Rightarrow \frac{M}{M_0} = e^{-V/u}$$

b) Ref. Foguete

$$M\, dv - \gamma_u\, dm\, u = 0 \qquad \gamma_u = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$M\, dv = -\mu\, dM$$

$$V + dV = \frac{V + dv}{1 + V\, dv/c^2} \quad \text{Ref. Lab.}$$

$$\hookrightarrow dv = \gamma^2 dV \longrightarrow \int \frac{dV}{1 - V^2/c^2} = \int -u\frac{dM}{M}$$

$$\Rightarrow \frac{M}{M_0} = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{c}{2u}} \simeq e^{-v/u}$$

Lista 4

1. $(t, r, \theta, \phi) \longrightarrow (T, \rho, \theta, \phi)$

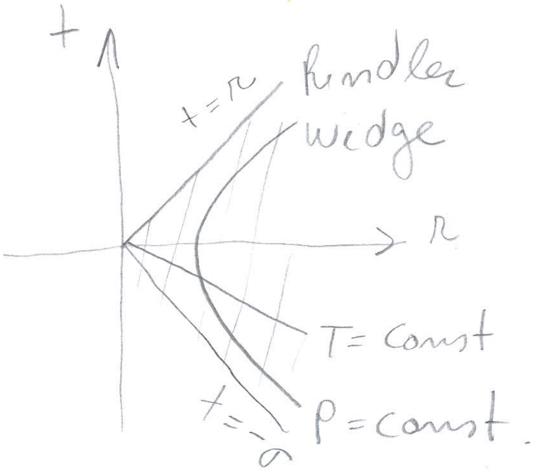
$$t = \rho\, \mathrm{sh}\, T \qquad dt = d\rho\, \mathrm{sh}\, T + \mathrm{ch}\, T\, \rho\, dT$$

$$r = \rho\, \mathrm{ch}\, T \qquad dr = d\rho\, \mathrm{ch}\, T + \mathrm{sh}\, T\, \rho\, dT$$

$$-dt^2 + dr^2 = d\rho^2 \underbrace{(-\mathrm{sh}^2 T + \mathrm{ch}^2 T)}_{=1} + dT^2 \underbrace{(-\rho^2 \mathrm{ch}^2 T + \rho^2 \mathrm{sh}^2 T)}_{=\rho^2}$$

$$+ d\rho\, dT(-2\rho\, \mathrm{sh}\, T\, \mathrm{ch}\, T + 2\rho\, \mathrm{ch}\, T\, \mathrm{sh}\, T)$$

$$\Rightarrow ds^2 = -\rho^2 dT^2 + d\rho^2 + \rho^2 \mathrm{ch}\, T (d\theta^2 + \sin^2\theta \, d\phi^2)$$



$$r^2 - t^2 = \rho^2$$

$$\Rightarrow \rho = \sqrt{r^2 - t^2}$$

$$-r \leq t \leq +r$$

Não cobre todo o espaço de Minkowski

(θ, ϕ) const.

5) $F_{\mu\nu} F^{\mu\nu}$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B & \dots \\ E_y & & 0 & \\ E_z & & & \end{pmatrix} \qquad T^{00} \to H$$

a)

$$= F_{00}F^{00} + \underbrace{F_{0i}}_{-E_i}\underbrace{F^{0i}}_{+E_i} + \underbrace{F_{i0}F^{i0}}_{F_{0i}F^{0i}} + \underbrace{F_{ij}F^{ij}}_{\epsilon_{ijk}B_k\epsilon^{ijk}B_k} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2)$$

b) $$\boxed{T^{\mu\nu} = F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4}\eta^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}}$$ Lista 4 ex. 8

$$T^{00} = F^{0}{}_{\lambda} F^{0\lambda} - \frac{1}{4}\eta^{00}(2\vec{E}^2 - 2\vec{B}^2)$$

$$= \vec{E}^2 + \frac{1}{2}(\vec{B}^2 - \vec{E}^2) = \frac{1}{2}(\vec{B} + \vec{E})$$

Ex 6)

$$\begin{cases} \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\times\vec{E} + \dfrac{\partial \vec{B}}{\partial t} = 0 \\ \vec{\nabla}\cdot\vec{E} = \cancel{\rho} = 0 \\ \vec{\nabla}\times\vec{E} - \dfrac{\partial \vec{B}}{\partial t} = \cancel{\vec{J}} = 0 \end{cases}$$

$$\vec{E} \to -\vec{B}$$

$$\partial_\mu F^{\mu\nu} = 0 \qquad F_{\mu\nu} \leftrightarrow \tilde{F}_{\mu\nu}$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

↳ Dualidade e/M para Maxwell sem fontes

→ Introduz monopolos magnéticos

$$\boxed{\begin{array}{l} \rho \leftrightarrow \rho_m \\ \vec{J} \leftrightarrow \vec{J}_m \end{array}}$$ Dirac 1927

$\boxed{\text{S-dualidade}}$

↳ Dualidade E/M é restabelecida