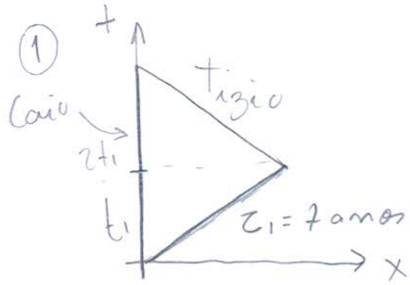
Relatividade - 23/05

$\boxed{\begin{array}{l} P_3 - 15/06 - \text{quarta} \\ \text{Sub} - 24/06 - \text{sexta} \end{array}}$

① 

$\tau_1 = 7$ anos (referencial próprio do Tizio)

$$t_1 = \tau_1 \frac{1}{\sqrt{1-\left(\frac{24}{25}\right)^2}} = 25 \text{ anos}$$

$$t_2 = 2t_1 = 50 \text{ anos}$$

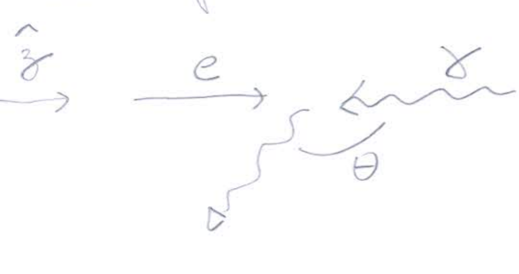
Caio $\rightarrow$ 75 anos

$$t_2 = 2t_1 \frac{1}{\sqrt{1-\left(\frac{12}{25}\right)^2}} = 44 \text{ anos}$$

Tizio $\Rightarrow \tau_1 + \tau_2 = 7 + 44 = 51$ anos

$75 - 51 = 24$ anos.

② Efeito compton "inverso"



$p^\mu = (E, 0, 0, p) \leftarrow$ elétron

$k^\mu = (\omega, 0, 0, -\omega) \leftarrow$ fóton antes

$k'^\mu = (\omega', 0, \omega' \sin\theta, \omega' \cos\theta)$ depois

($p^{\mu'}$)

Conservação

$$p^\mu + K^\mu = p'^\mu + K'^\mu$$

$$p^{\mu'} = p^\mu + K'^\mu - K^\mu$$

$$(p^{\mu'})^2 = -m^2 = (p^\mu + K^\mu - K^{\mu'})^2$$

$$= -m^2 + 2p.K - 2p.K' - 2K.K'$$

$$K.K' = p.K = p.K - p.K'$$

$$\omega' = \frac{\omega(E+p)}{E+\omega-(p-\omega)\cos\theta}$$

$\omega \ll p \rightarrow p-\omega > 0$

$\cos\theta = 1$

$\boxed{\theta = 0}$

backscattering

$$\omega'_{max} = \frac{\omega(E+p)}{E+\omega-p+\omega}$$

$P = \sqrt{E^2-m^2} \simeq E - \frac{m^2}{2E}$

componente z de p^μ

$$= \frac{E}{1+m^2/4\omega_E}$$

③
$$\begin{cases} t = \left(\frac{c}{g} + \frac{x'}{c}\right)\sinh\left(\frac{gt'}{c}\right) \\ x = c\left(\frac{c}{g} + \frac{x'}{c}\right)\cosh\left(\frac{gt'}{c}\right) - \frac{c^2}{g} \end{cases}$$

$$dt = \frac{dx'}{c}\,sh\left(\frac{gt'}{c}\right) + \frac{g}{c}\left(\frac{c}{g} + \frac{x'}{c}\right)ch\left(\frac{gt'}{c}\right)dt'$$

$$dx = dx'\,ch\left(\frac{gt'}{c}\right) + g\left(\frac{c}{g} + \frac{x'}{c}\right)sh\left(\frac{gt'}{c}\right)dt'$$

$dy' = dy$

$dz' = dz$

$$-c^2dt^2 + dx^2 = \begin{cases} dt'^2 \quad \left(-g^2\left(\frac{c}{g}+\frac{x'}{c}\right)\right. \\ dx'^2 \quad (1) \\ dt'dx' \quad (0) \end{cases}$$

a)
$$\boxed{= -c^2\left(1+\frac{gx'}{c^2}\right)^2 dt'^2 + dx'^2 + dy'^2 + dz'^2}$$

$gt' << c$

b) $t \simeq t' \quad x \simeq x' + \frac{1}{2}gt'^2 = x' + \frac{1}{2}gt^2$

c) $(d\tau)_{x'=0} \overset{c=1}{=} dt'$

$$(d\tau)_{x'=h} = dt'\left(1+\frac{gh}{c^2}\right)$$

$$\boxed{(d\tau)_{x'=z} = \left(1+\frac{gh}{c^2}\right)(d\tau)_{x'=0}}$$

Redshift em um campo gravitacional

4 $ds^2 = 0$

$$\begin{cases} ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu = 0 \\ ds^2 = \Omega^2 \underbrace{g_{\mu\nu}\,dx^\mu dx^\nu}_{=0} = 0 \end{cases}$$

$\Rightarrow$

5 a) $A_\mu = (\phi, \vec{A})$

$$\frac{d\vec{p}}{dt} = -\frac{q}{c}\frac{\partial \vec{A}}{\partial t} - q\vec{\nabla}\phi - \frac{q}{c}\vec{v}\times(\vec{\nabla}\times\vec{A})$$

$$= q\vec{E} + \frac{q}{c}\vec{v}\times\vec{B}$$

$$\vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

1D $\quad E_x \neq 0 \qquad \boxed{\frac{dp}{dt} = qE}$

b) $v = \frac{dx}{dt} = \frac{dx/d\tau}{dt/d\tau} \Rightarrow \frac{dx}{d\tau} = c\, sh\left(\frac{qE\tau}{mc}\right)$

$$\frac{dt}{d\tau} = ch\left(\frac{qE\tau}{mc}\right)$$

$\hookrightarrow v = c\ th\left(\frac{qE\tau}{mc}\right)$

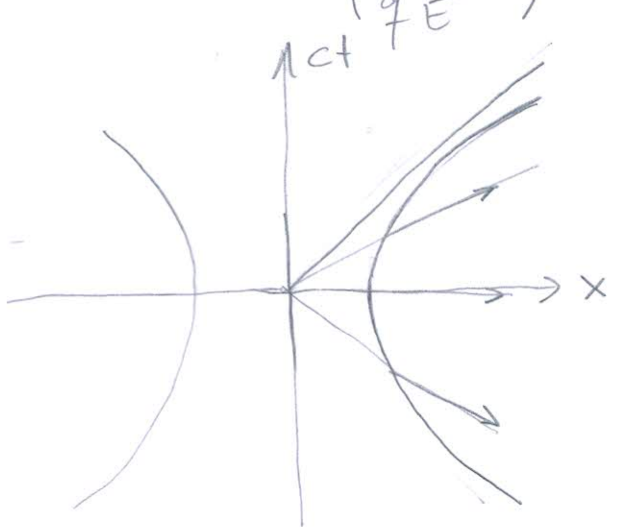
$1 - th^2 = \frac{1}{ch^2} \qquad p = \frac{mc\, th()}{\frac{1}{ch()}} = mc\, sh(\ )$

$\frac{dp}{dt} = \frac{dp/d\tau}{dt/d\tau} = \frac{qE ch()}{ch()} = qE$ ✓

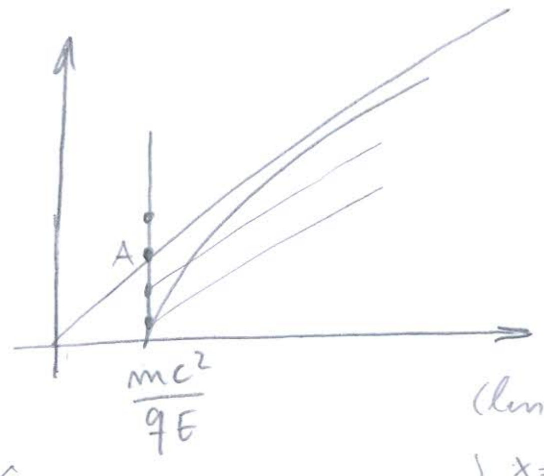
c) $w^\mu = \frac{1}{m}\frac{dp^\mu}{d\tau}$

$w^\mu w_\mu = -\left(\frac{qE}{m}\right)^2$

$x^2 - c^2t^2 = \left(\frac{mc^2}{qE}\right)^2$ hipérbole



$\rightarrow$



linha vertical

$\hookrightarrow$ Interseção de $\left\{ \begin{array}{l} x = \frac{mc^2}{qE} \\ x = ct \end{array} \right.$

$x \rightarrow t = \frac{mc}{qE}$

(6) $R_{\mu\nu\rho\sigma}$ 1,2

tenho $g_{\mu\nu}$

$D_\mu \cancel{g_{\mu\nu}} = 0$

$\cancel{R_{\mu\nu}}$

$\partial_\mu \cancel{g_{\rho\nu}}$ não é um tensor

$$R_{1212} = R_{2121} = -R_{1221} \cdots$$

$R_{\tau\rho\mu\nu}$ tem que ter as simetrias certas

$$\boxed{R_{\tau\rho\mu\nu} = f(R)\left(g_{\tau\mu} g_{\rho\nu} - g_{\tau\nu} g_{\rho\mu}\right)}$$

$$g^{\tau\mu} R_{\tau\rho\mu\nu} = f(R)\left(2 g_{\rho\nu} - \delta^\mu_\nu g_{\rho\mu}\right) = R_{\rho\nu}$$

$$g^{\rho\nu} R_{\rho\nu} = f(R)(2.2 - 2) = 2 f(R) = R$$

$$\rightarrow \boxed{f(R) = \frac{R}{2}}$$

$$\left[R_{\tau\rho\mu\nu} = \frac{R}{2}\left(g_{\tau\mu} g_{\rho\nu} - g_{\tau\nu} g_{\rho\mu}\right)\right] g^{\rho\nu}$$

$$\Rightarrow R_{\tau\mu} = g_{\tau\mu} \frac{R}{2}$$

$$R_{\tau\mu} - g_{\tau\mu}\frac{R}{2} = 0$$

Gravidade em 2D não é dinâmica

$$\boxed{\int d^2x \sqrt{-g}\, R}$$ Gauss-Bonet