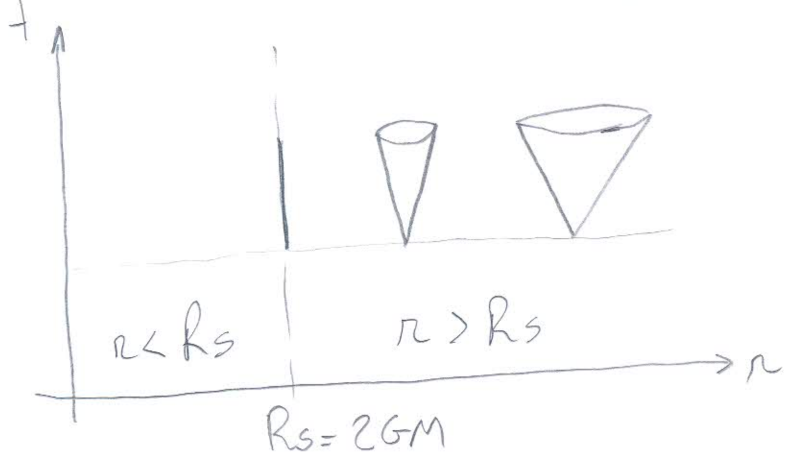
## Relatividade - 25/05

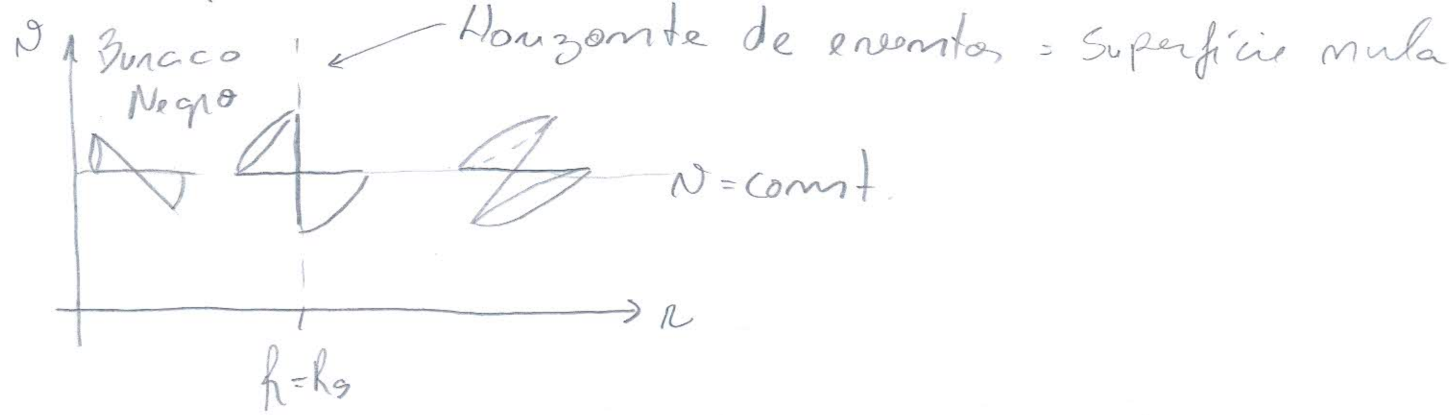
Schwarzschild

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2 d\Omega_2^2$$




E-F

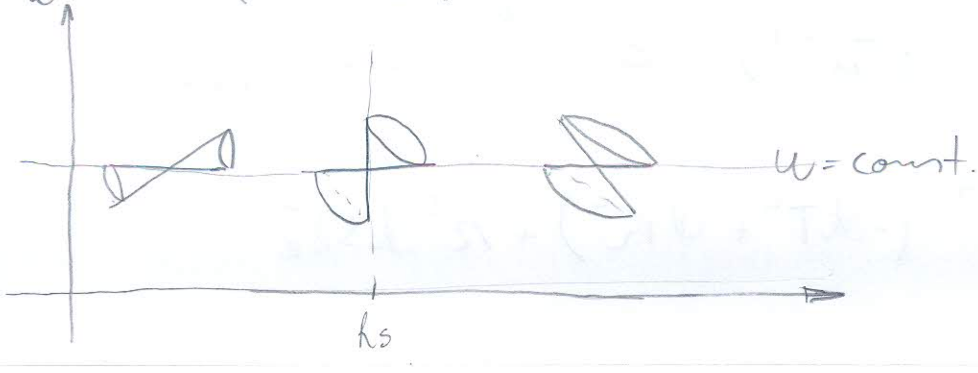
$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dv^2 + 2dv\,dr + r^2 d\Omega_2^2$$




Extensões? Sim

Por exemplo, poderíamos ter usado $(u, r)$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)du^2 - 2du\,dr + r^2 d\Omega_2^2$$



$(u, v)$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) du\, dv + r^2 d\Omega_2^2$$

$$r = r(u, v)$$

com $\boxed{\frac{1}{2}(v-u) = r + 2GM \ln\left(\frac{r}{2GM} - 1\right)}$

degenerescência

$$r = R_S : \begin{cases} v = -\infty \\ \text{ou} \\ u = +\infty \end{cases}$$

$$\begin{cases} v' = e^{v/4GM} = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{\frac{r+t}{4GM}} \\ u' = e^{-u/4GM} = -\left(\frac{r}{2GM} - 1\right)^{1/2} e^{\frac{r-t}{4GM}} \end{cases}$$

$$ds^2 = -\frac{16G^3M^3}{r} e^{-r/2GM} (2du'\, dv') + r^2 d\Omega_2^2$$

$r = R_S$ <u>OK</u>

$u', v'$ : coordenadas de tipo nulo

$\theta, \phi$ : Coordenadas tipo espaço

→ Seria melhor ter uma coordenada de tipo tempo e 3 de tipo-espaço

$$\begin{cases} T = \frac{1}{2}(v' + u') = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{r/4GM} \operatorname{sh}\left(\frac{t}{4GM}\right) \\ R = \frac{1}{2}(v' - u') = \left(\frac{r}{2GM} - 1\right)^{1/2} e^{r/4GM} \operatorname{ch}\left(\frac{t}{4GM}\right) \end{cases}$$

$$ds^2 = \frac{32G^3M^3}{r} e^{-r/2GM} (-dT^2 + dR^2) + r^2 d\Omega_2^2$$

$r = r(R, T)$ com $T^2 - R^2 = \left(1 - \frac{r}{2GM}\right) e^{r/2GM}$

Propriedades:

1) Curvas nulas radiais são sempre a 45°

$$\underbrace{\left.\begin{matrix}\theta = \text{const} \\ \phi = \text{const}\end{matrix}\right| \longrightarrow d\Omega_2^2 = 0}$$

$$ds^2\Big|_{\substack{\theta=\text{const}\\ \phi=\text{const}}} = 0 \qquad \boxed{T = \pm R + \text{const}}$$

2) $r = R_s$ é infinitamente longe

$T = \pm R$

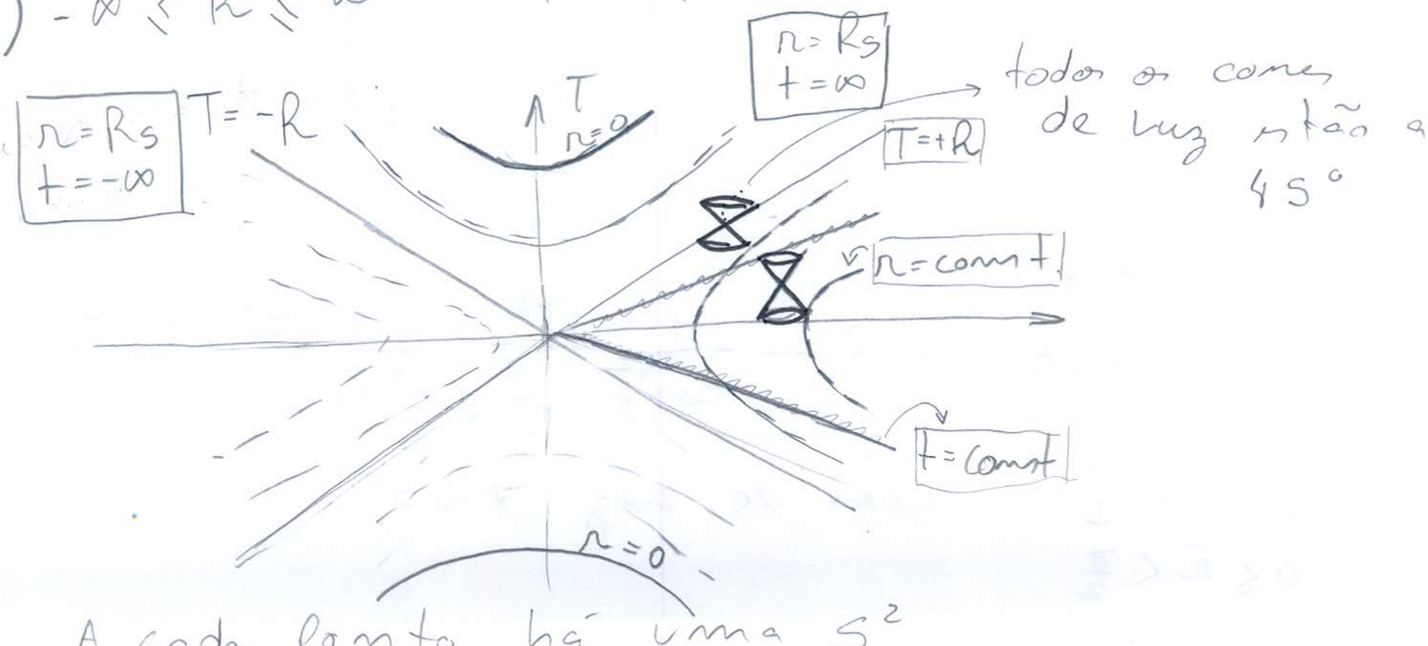
3) $r = \text{const.} \longrightarrow T^2 - R^2 = \text{const} \rightarrow$ hiperbólica

$t = \text{const} \rightarrow \frac{T}{R} = th\left(\frac{t}{4GM}\right)$ linhas retas passando pela origem com "slope" $th(t/4GM)$

$t \to \pm\infty \quad \frac{T}{R} = \pm 1$

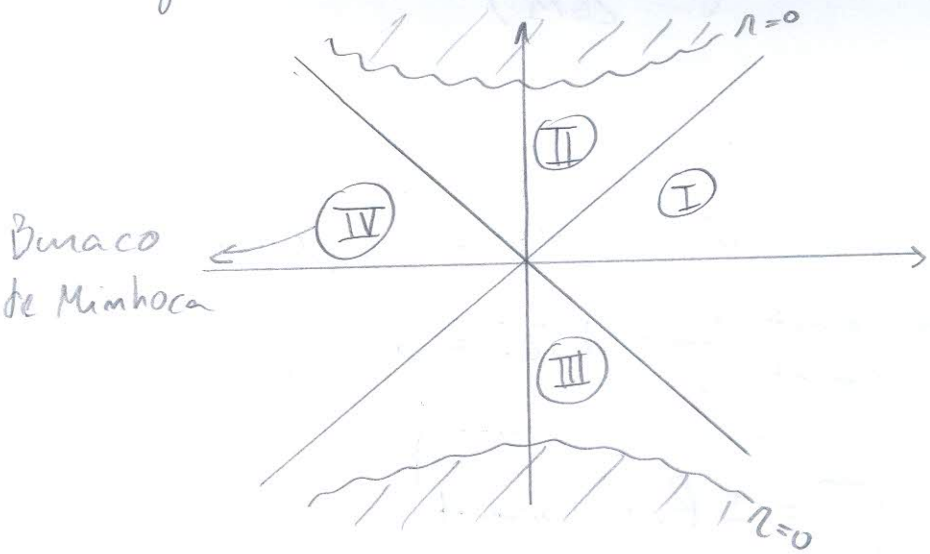
$t = \pm\infty$ é a mesma superfície de $r = R_s$

4) $-\infty \leq R \leq \infty \qquad T^2 < R^2 + 1$




A cada ponto há uma $S^2$

# Diagrama de Kruskal




(I) Schwarzschild
$r > 2GM = R_S$

(II): Buraco Negro

(III): Buraco Branco
(não é físico)

(IV): É uma cópia de (I) sem contato causal

## Diagramas Conformes ou de Penrose ou de Carter-Penrose

*) Cone de luz a 45°

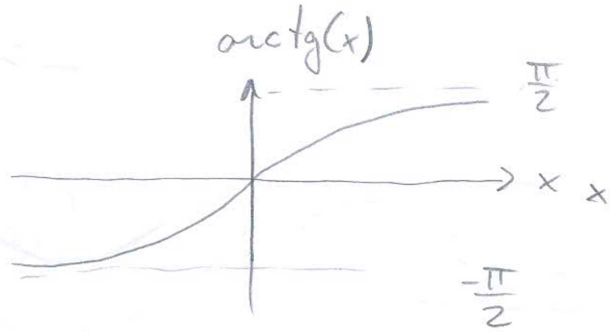
*) "Infinitos à distância finita"

1) Minkowski

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2$$

Cone de luz: 45° ✓

$$\begin{cases} -\infty < t < \infty \\ 0 \leq r < \infty \end{cases}$$

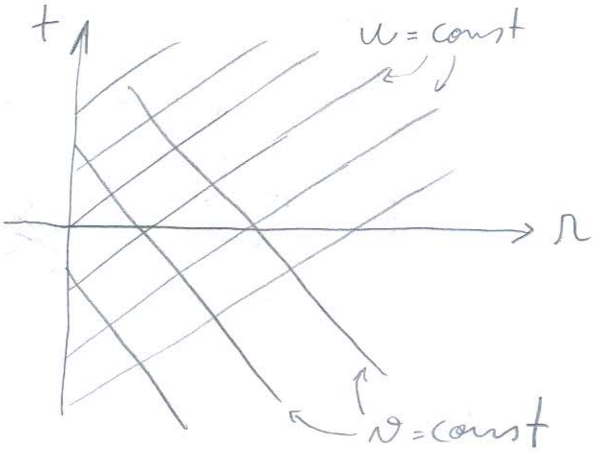



$$\begin{cases} \bar{t} = \operatorname{arctg} t \\ \bar{r} = \operatorname{arctg} r \end{cases} \qquad ds^2 = -\frac{1}{\cos^4 \bar{t}} d\bar{t}^2 + \frac{1}{\cos^4 \bar{r}} d\bar{r}^2 + \operatorname{tg}^2 \bar{r}\, d\Omega_2^2$$

$$\begin{cases} -\frac{\pi}{2} < \bar{t} < \frac{\pi}{2} \\ 0 \leq \bar{r} < \frac{\pi}{2} \end{cases} \qquad \text{Cone de luz} \neq 45°$$

$$\frac{d\bar{t}}{d\bar{r}} = \pm \frac{\cos^2 \bar{t}}{\cos^2 \bar{r}} \neq \pm 1$$

$$\longrightarrow \quad u = t - r \qquad v = t + r \qquad \begin{cases} -\infty < u < \infty \\ -\infty < v < \infty \end{cases} \quad u \leq v$$




$$ds^2 = -\frac{1}{2}(du\,dv + dv\,du) + \frac{1}{4}(v-u)^2 d\Omega_2^2$$

$$\begin{cases} U = \operatorname{arctg} u \\ V = \operatorname{arctg} v \end{cases} \longrightarrow \begin{cases} -\pi/2 < U < \pi/2 \\ -\pi/2 < V < \pi/2 \end{cases} \quad U \leq V$$

$$ds^2 = \frac{1}{4\cos^2 U \cos^2 V}\left[-2(dU\,dV + dV\,dU) + \sin^2(V-U)\,d\Omega_2^2\right]$$

$$\hookrightarrow \begin{cases} T = V + U \\ R = V - U \end{cases} \rightarrow \begin{cases} 0 \leq R \leq \pi \\ |T| + R < \pi \end{cases}$$

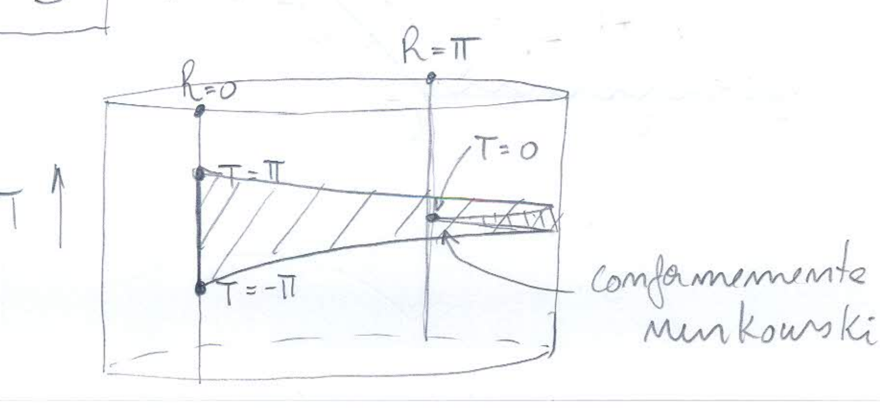
$$ds^2 = w^{-2}(T,R)\left(-dT^2 + dR^2 + \sin^2 R\, d\Omega_2^2\right)$$

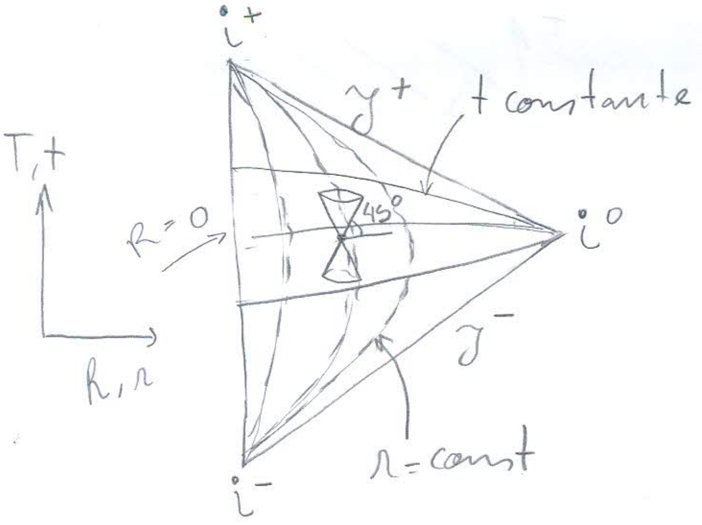
$$w = \cos T + \cos R$$

$$\boxed{d\tilde{s}^2 = -dT^2 + dR^2 + \sin^2 R\, d\Omega_2^2} = w^2 ds^2$$

Métrica não física

$\boxed{\mathbb{R} \times S^3}$ → Universo estático de Einstein






$i^+$ = infinito futuro de tipo tempo
($T=\pi$ , $R=0$)

$i^0$ = infinito espacial
($T=0$ , $R=\pi$)

$i^-$ = infinito passado de tipo tempo
($T=-\pi$ , $R=0$)

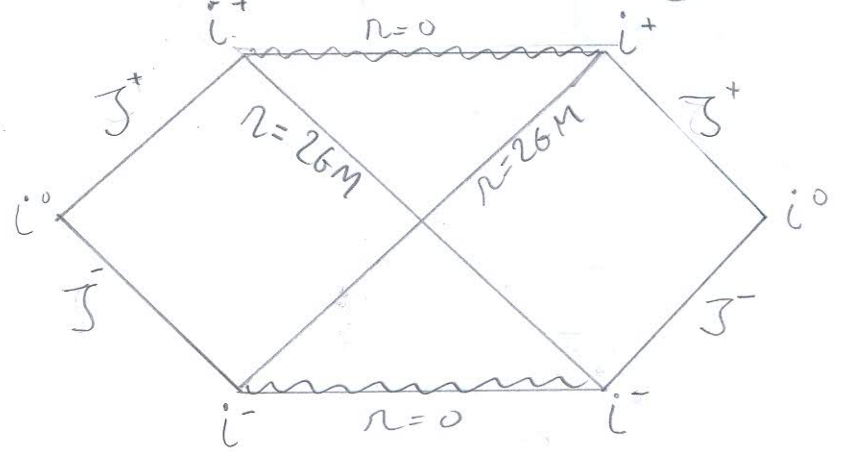
$\mathcal{J}^+$ = infinito futuro de tipo nulo
$\begin{pmatrix} T=\pi-R \\ 0<R<\pi \end{pmatrix}$

$\mathcal{J}^-$ = inf. passado nulo $\begin{pmatrix} T=-\pi+R \\ 0<R<\pi \end{pmatrix}$

Schwarzschild

$$ds^2 = -\frac{16GM^3}{r} e^{-r/4GM} \left(2dv'du'\right) + r^2 d\Omega_2^2$$

$$\begin{cases} v'' = \operatorname{arctg}\left(\dfrac{v'}{\sqrt{2GM}}\right) \\ u'' = \operatorname{arctg}\left(\dfrac{u'}{\sqrt{2GM}}\right) \end{cases} \qquad \begin{cases} -\frac{\pi}{2} < v'' < \pi/2 \\ -\frac{\pi}{2} < u'' < \pi/2 \\ -\frac{\pi}{2} < v'' + u'' < \pi/2 \end{cases}$$




$\boxed{i^\pm \neq r=0}$