Física matemática I — FMA204
Prova substitutiva — 04/julho/2001

1. Obtenha a série de Fourier de uma função de período 3 e que satisfaz

$$f(x) = \begin{cases} x & \text{para } 0 \leq x < \frac{3}{2} \\ 0 & \text{para } \frac{3}{2} \leq x < 3 \end{cases}$$

   Para que valor a série deveria convergir no ponto $\frac{3}{2}$?

   Valor 2,0

2. Resolva o seguinte problema de condução de calor bidimensional:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \kappa \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] && \text{em} \quad R \ , \\ u(0,y,t) &= u(L_x,y,t) = 0 && \text{para} \quad t > 0 \\ \frac{\partial u}{\partial y}(x,0,t) &= u(x,L_y,t) = 0 && \text{para} \quad t > 0 \\ u(x,0) &= xy && \text{para} \quad 0 < x < L_x \ ; \ 0 < y < L_y \ . \end{aligned}$$

   Valor 4,0

3. Use a transformada de Fourier para resolver o seguinte problema:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \kappa \frac{\partial^2 u}{\partial x^2} \quad \text{em} \quad t > 0 \ , \ x \in R \ , \\ u(x,0) &= \delta(x-a) \quad \text{para} \quad x \in R \ . \end{aligned}$$

   Valor 3,0

4. Resolva formalmente o seguinte problema usando separação de variáveis:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= c^2 \frac{\partial^2 u}{\partial x^2} - 2\gamma \frac{\partial u}{\partial t} && \text{em} \quad t > 0 \ , \ 0 < x < L \ , \\ u(0,t) &= u(L,t) = 0 && \text{para} \quad t > 0 \\ u(x,0) &= f(x) && \text{para} \quad 0 < x < L \ , \\ \frac{\partial u}{\partial t}(x,0) &= 0 && \text{para} \quad 0 < x < L \ . \end{aligned}$$

   Assuma que $\gamma < \pi c/L$.

   Valor 2,0

Algumas fórmulas:

$$\Gamma(z) = \int_0^\infty dt e^{-t} t^{z-1} \quad , \quad \mathcal{F}[e^{-\alpha x^2}] = \frac{e^{-\xi^2/4\alpha}}{\sqrt{2\alpha}} \quad ,$$

$$\int dt\; t \sin(at) = \frac{\sin(at)}{a^2} - \frac{t\cos(at)}{a} \quad , \quad \int dt\; t\cos(at) = \frac{\cos(at)}{a^2} + \frac{t\sin(at)}{a}$$

$$\frac{dx}{dt} = -\alpha x + f(t) \Longrightarrow x(t) = x(0)e^{-\alpha t} + e^{-\alpha t}\int_0^t ds\; e^{\alpha s} f(s) \quad ,$$

$$\frac{f(x+0)+f(x-0)}{2} = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

$$a_n = \frac{1}{L}\int_{-L}^{L} dx f(x)\cos\left(\frac{n\pi x}{L}\right) \quad ; \quad b_n = \frac{1}{L}\int_{-L}^{L} dx f(x)\sin\left(\frac{n\pi x}{L}\right)$$

$$\frac{1}{L}\int_{-L}^{L} dx |f(x)|^2 = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$$