Apêndice F

Relação entre magnitude de velocidades em S e S′

Da transformação de Lorentz inversa (de S′ para S) para velocidades, temos

$$V_x = \frac{V'_x + v}{1 + \frac{vV'_x}{c^2}} \tag{F.1}$$

$$V_y = \frac{V'_y}{\gamma(1 + \frac{vV'_x}{c^2})} \qquad \text{(Transformação de Velocidades - Inversa)} \tag{F.2}$$

$$V_z = \frac{V'_z}{\gamma(1 + \frac{vV'_x}{c^2})} \tag{F.3}$$

onde o sistema S′ se move com velocidade v com respeito a S. Definindo $V^2 = V_x^2 + V_y^2 + V_z^2$, temos

$$V^2 = \frac{(V'_x + v)^2 + (V'_y)^2(1 - v^2/c^2) + (V'_z)^2(1 - v^2/c^2)}{(1 + \frac{vV'_x}{c^2})^2} \tag{F.4}$$

Portanto

$$\begin{aligned} 1 - \frac{V^2}{c^2} &= 1 - \frac{(V'_x + v)^2/c^2 + (V'_y)^2/c^2(1 - v^2/c^2) + (V'_z)^2/c^2(1 - v^2/c^2)}{(1 + \frac{vV'_x}{c^2})^2} \\ &= \frac{(1 + \frac{vV'_x}{c^2})^2 - (V'_x + v)^2/c^2 - (V'_y)^2/c^2(1 - v^2/c^2) - (V'_z)^2/c^2(1 - v^2/c^2)}{(1 + \frac{vV'_x}{c^2})^2} \end{aligned} \tag{F.5}$$

Portanto

$$\begin{aligned} (1 - \frac{V^2}{c^2})(1 + \frac{vV'_x}{c^2})^2 &= 1 + \frac{2vV'_x}{c^2} + \frac{v^2(V'_x)^2}{c^4} - \frac{(V'_x)^2}{c^2} - \frac{2V'_x v}{c^2} - \frac{v^2}{c^2} - (\frac{(V'_y)^2}{c^2} + \frac{(V'_z)^2}{c^2})(1 - \frac{v^2}{c^2}) \\ &= 1 - \frac{(V'_x)^2}{c^2}(1 - \frac{v^2}{c^2}) - \frac{v^2}{c^2} - (\frac{(V'_y)^2}{c^2} + \frac{(V'_z)^2}{c^2})(1 - \frac{v^2}{c^2}) \\ &= 1 - \frac{v^2}{c^2} - (\frac{(V'_x)^2}{c^2} + \frac{(V'_y)^2}{c^2} + \frac{(V'_z)^2}{c^2})(1 - \frac{v^2}{c^2}) \\ &= (1 - \frac{v^2}{c^2}) - \frac{(V')^2}{c^2}(1 - \frac{v^2}{c^2}) \\ &= (1 - \frac{v^2}{c^2})(1 - \frac{(V')^2}{c^2}) \end{aligned} \tag{F.6}$$

ou

$$\boxed{1-\frac{V^2}{c^2}=\frac{\left(1-\frac{(V')^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{vV'_x}{c^2}\right)^2}} \tag{F.7}$$

De forma similar, tem-se

$$1-\frac{(V')^2}{c^2}=\frac{\left(1-\frac{V^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{vV'_x}{c^2}\right)^2} \tag{F.8}$$