PGF5292: Physical Cosmology I

Problem Set 4

(Due April 23, 2021)

1) Dodelson 2.4

2) Dodelson 2.8

3) Dodelson 2.10 (You may want to solve problem 7 below first).

4) Dodelson 2.11

5) Dodelson 2.13

6) Dodelson 2.18

7) Scale Factor Evolution (worth 4 problems): In this problem, you will solve the Friedmann equation **numerically**.

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\underbrace{\Omega_k a^{-2}}_{\text{Curvature}} + \underbrace{\Omega_\mathrm{m} a^{-3}}_{\text{Matter}} + \underbrace{\Omega_\mathrm{r} a^{-4}}_{\text{Radiation}} + \underbrace{\Omega_\mathrm{DE} a^{-3(1+w)}}_{\text{Dark Energy}}\right] \quad (1)$$

$$\text{where} \quad \Omega_k = 1 - (\Omega_\mathrm{m} + \Omega_\mathrm{r} + \Omega_\mathrm{DE}) \quad (2)$$

For that, you must write a numerical program that uses a differential equation solver (e.g. Runge-Kutta). I **highly** suggest you use something like Python, C/C++ or Fortran, since it will make it easier to integrate with other cosmological codes in the future. If you don't know any of these languages, it is a good time to learn. You can then try to find free efficient numerical solvers for differential equations to include into your program.

In each case below, set up appropriate **initial conditions** using the dominant component at early times. For instance, for a universe with matter and radiation, at early times radiation dominates and you may find the analytical solutions $a(t)$ to set the correct value of $a_0 = a(t_0)$ at the initial time t_0.

Note that the only quantity with units here is H_0 (units of time^{-1}). Use H_0 such that you present your results with time in Gigayears (Gyr). For all cases, fix $h = 0.72$.

a) First do the **single-component** cases. Leave one Ω_i at a time, and set all others equal to zero. For the dark energy, choose a cosmological constant, i.e. $w = -1$. For each case, plot the numerically derived scale factor as a function of time and compare your numerical solution to the analytical solution, plotting also the analytical solution.

b) Now do an intermediate **two-component** case, containing matter + cosmological constant. In this case you can also find an analytical solution to compare.

c) Now do the **complete** case with all terms. Use **fiducial** values: $\Omega_k = 0$ (flat universe), $\Omega_{\rm m} = 0.25$, $\Omega_{\rm DE} = 0.75$, $w = -1$, $\Omega_{\rm r} = 8.2 \times 10^{-5}$. You can use a more precise value of $\Omega_{\rm r}$, computing ρ_γ (Dodelson Eq. 2.69) for $T_\gamma = 2.725$ K, adding ρ_ν for 3 massless neutrinos (Dodelson Eq. 2.77), and dividing by $\rho_{\rm crit}$. Compute the **age of the Universe**, by finding the time that corresponds to today, i.e. the time when $a(t = \text{today}) = 1$. Change the parameter values one at a time (Ω's and w for cases below) and **plot** the corresponding $a(t)$ for each variation set. See also the impact on the age of the Universe.

i) $\Omega_{\rm m} = 0.1, 0.25, 0.5, 1.0$ (with all other parameters equal to fiducial)

ii) $\Omega_{\rm DE} = 0.5, 0.75, 1.0$ (with all other parameters equal to fiducial)

iii) $w = -0.8, -1.0, -1.2$ (with all other parameters equal to fiducial)

iv) $\Omega_{\rm r} = (6, 8, 10) \times 10^{-5}$ (with all other parameters equal to fiducial)

v) Flat cases: $(\Omega_{\rm m}, \Omega_{\rm DE}) = (0.0, 1.0), (0.1, 0.9), (0.25, 0.75), (0.75, 0.25), (1.0, 0.0)$.

Indicate clearly in the plots, the cases you are showing.