PGF5299: Physical Cosmology II

Problem Set 3

(Due September 28, 2021)

1) Correlation Function in Redshift Space. In class, we saw that the correlation function in redshift space is anisotropic in space and given by

$$\xi_g^s(\mathbf{s}) = \xi_g^s(s_\perp, s_\parallel) = \xi_g^s(s, \mu_s) = b^2 \sum_{\ell=0,2,4} c_l(\beta) L_\ell(\mu_s)\ \xi_\ell^s(s) \tag{1}$$

where $L_\ell(\mu_s)$ is the Legendre Polynomial of order ℓ, $\mu_s = \cos(\theta_s)$ is the cosine of the angle between the position vector $\mathbf{s}$ and the line-of-sight $\hat{\mathbf{z}}$, the coefficients are

$$c_\ell(\beta) = \frac{2\ell+1}{2}\int_{-1}^{1}(1+\beta x^2)^2 L_\ell(x)dx = \begin{cases} 1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2, & \ell = 0 \\ \frac{4}{3}\beta + \frac{4}{7}\beta^2, & \ell = 2 \\ \frac{8}{35}\beta^2, & \ell = 4 \end{cases} \tag{2}$$

where $\beta = f/b$, b is the galaxy bias, $f = \frac{d\ln D}{d\ln a}$ and the multipoles are

$$\xi_\ell^s(s) = i^\ell \int \frac{k^2 dk}{2\pi^2} j_\ell(ks) P^r(k) \tag{3}$$

a) From the real-space matter power spectrum $P^r(k)$ (e.g. from CAMB), use Eq. (3) to compute the multipoles $\xi_\ell^s(s)$ for $\ell = 0, 2, 4$. Plot them in log-scale and appropriate ranges.

b) Use your numerical growth function $D(z)$ to compute $f(z)$. Compare to fitting function $f = \omega_m^\gamma(z)$, $\gamma \approx 0.55$. [1] Set $b = 1$ and use Eq. (2) to compute coefficients c_ℓ vs. z.

c) Finally, use Eq. (1) to obtain $\xi_g^s(s_\perp, s_\parallel)$ as a function of perpendicular and parallel coordinates at $z = 0$ and 1.0 and make a 2D plot of your results, with $s_\perp = s\sin\theta_s$ in the x-axis and $s_\parallel = s\cos\theta_s$ in the y-axis, and a color code for the value of $\xi_g^s(s_\perp, s_\parallel)$. Compare these results to a similar 2D plot for the isotropic real space correlation $\xi^r(s)$.

[1] Here $\omega_m(z) = (1+z)^3\Omega_m/E^2(z)$ and $E^2(z) = (1+z)^3\Omega_m + \Omega_\Lambda$ for a flat cosmology.

2) Multipoles and Bessel Function. Use Eq. (3) and the properties of spherical Bessell functions $j_\ell(x)$ to show that the multipoles can also be written in terms of the real-space correlation function $\xi^r(x)$ as

$$\begin{aligned}
\xi_0^s(s) &= \int \frac{k^2 dk}{2\pi^2} \frac{\sin(ks)}{ks} P^r(k) = \xi^r(s) \\
\xi_2^s(s) &= \left(\frac{3}{s^3} \int_0^s dx\ x^2 \xi^r(x) \right) - \xi^r(s) \\
\xi_4^s(s) &= \frac{5}{2} \left(\frac{3}{s^3} \int_0^s dx\ x^2 \xi^r(x) \right) - \frac{7}{2} \left(\frac{5}{s^5} \int_0^s dx\ x^4 \xi^r(x) \right) + \xi^r(s)
\end{aligned}$$

This is useful to propagate into redshift space, non-linear prescriptions that modify the real-space power spectrum or correlation. For instance, from a prescription $\xi^r(x) \to \xi^r_{\rm NL}(x)$ that helps improve the BAO peak by accounting for non-linear effects [2], one can obtain further $\xi^s_{\rm NL}(\mathbf{s})$. Notice that

$$\bar{\xi}^r(s) = \frac{3}{s^3} \int_0^s dx\ x^2 \xi^r(x) = \frac{3}{4\pi s^3} \int_0^s d^3x\ \xi^r(x)$$

represents the *volume average* of ξ^r up to radius s.

[2] See e.g. Crocce and Scoccimarro 2008, arxiv:0704.2783, Fig. 1.