PGF5299: Physical Cosmology II

Problem Set 5

(Due October 22, 2021)

1) Halo Model Ingredients

In PS4, you computed the halo mass-function for a fit from (Tinker et al. 2008) as described in the main text of this paper. Now let us consider Tinker's alternative fit

$$\frac{dn(z, M_{vir})}{d\ln M_{vir}} = g(\sigma)\frac{\bar{\rho}_{m0}}{M_{vir}}\frac{d\ln\sigma^{-1}}{d\ln M_{vir}} \tag{1}$$

$$g(\sigma) = B\left[\left(\frac{\sigma}{e}\right)^{-d} + \sigma^{-f}\right]\exp\left[-\frac{g}{\sigma^2}\right] \tag{2}$$

where instead we use the function $g(\sigma)$ from Appendix C in Tinker's paper (why do we do this?). Set parameter values for parameters B, d, e, f, g which are appropriate for $\Delta = \Delta_c/\Omega_m$ (computed from Eqs. 10-12 below, which sets the mass to be the virial mass $M = M_{vir}$). For $\Omega_m = 0.23$, $\Delta \approx 397$, which you may approximate as $\Delta = 400$ for purposes of using Tinker's fits. Here $\sigma^2(M_{vir})$

$$\sigma^2(z, R) = G^2(z)\int\frac{k^2dk}{2\pi^2}|W(kR)|^2P_L(k) \tag{3}$$

where $M_{vir} = \bar{\rho}_{m0}4\pi R^3/3$, $\bar{\rho}_{m0} = \Omega_m\rho_{crit,0}$, $P_L(k)$ is the linear power spectrum at redshift $z = 0$, $G(z) = D(z)/D(z = 0)$ is the growth function normalized to its value today, and

$$W(kR) = \frac{3}{k^2R^2}\left[\frac{\sin(kR)}{kR} - \cos(kR)\right] \tag{4}$$

is the Fourier Transform of a spherical top-hat window of radius R.

a) For a fiducial cosmology, compute the halo mass-function above and plot it versus M_{vir} for $z = 0$ and $z = 1$ in the range $M_{vir} = [10^{12}, 10^{16}]M_\odot/h$. Use log scale in both axes.

Check that the mass function is properly normalized i.e. that $\int[-g(\sigma)/\sigma]d\sigma = 1$. If you do this integral from 0 to σ_{max}, for what value of σ_{max} the integral is equal to 0.95? What value of M_{vir} corresponds to this σ_{max}? This means that if you do integrations in halo mass with this lower limit mass, you're missing out 5% of the background energy density.

b) For the fiducial cosmology, compute the halo bias for the fit from Tinker et al. 2010:

$$b(z, M) \quad = \quad 1 - A\frac{\nu^a}{\nu^a + \delta_c^a} + B\nu^b + C\nu^c \tag{5}$$

where $\nu = \delta_c/\sigma$. Again, for consistency use values of A, a, B, b, C, c which are appropriate for $\Delta = \Delta_c/\Omega_m$ (≈ 400 for $\Omega_m = 0.23$), i.e. $M = M_{vir}$. See Tinker's Eqs. 6 and 7 and Table 2. Notice that the paper for Tinker's bias (2010) is different from that for the mass-function (2008)!

Plot $b(z, M_{vir})$ versus M_{vir} for $z = 0$ and $z = 1$ in the range $M_{vir} = [10^{12}, 10^{16}]M_\odot/h$. Use log scale in the M_{vir} axis and *linear* scale for $b(M_{vir})$.

Check that the bias function above is properly normalized, i.e. $\int[g(\nu)b(\nu)/\nu]d\nu = 1$. If the mass-function and/or bias are *not* normalized appropriately, you will likely obtain incorrect results for the 2-halo term in Eq. 19 below (which will not properly approach the linear spectrum at large scales). The 1-halo term in Eq. 18 will also be affected.

c) For the fiducial cosmology, compute the halo profile for the fit of NFW:

$$\rho(r|M_{vir}, z) = \frac{\rho_s}{cr/r_{vir}(1 + cr/r_{vir})^2} \tag{6}$$

for halos of mass $M_{vir} = 10^{14}$ and $10^{15}M_\odot/h$, at redshifts $z = 0$ and $z = 1$. Plot $\rho(r)$ versus r for these 4 cases in the same plot with log scale in both axes. Notice that ρ_s, c and r_{vir} are functions of M_{vir} and z. In order to determine these 3 quantities for a given M_{vir} and z, you need the formulae sequence below:

$$
\begin{aligned}
E^2(z) &= \Omega_m(1+z)^3 + \Omega_{DE}(1+z)^{3(1+w)}\,, && (7)\\
\rho_{crit,0} &= 2.775\times 10^{11}\ h^2 M_\odot \mathrm{Mpc}^{-3}\,, && (8)\\
\rho_{crit}(z) &= \rho_{crit,0}E^2(z)\,, && (9)\\
\omega_m(z) &= \Omega_m(1+z)^3/E^2(z)\,, && (10)\\
x &= \omega_m(z)-1\,, && (11)\\
\Delta_c &= 18\pi^2+82x-39x^2\,, \quad (\text{Bryan \& Norman 1997}) && (12)\\
\Delta_c &= \frac{3M_{vir}}{\rho_{crit}(z)4\pi r_{vir}^3} \quad \rightarrow \quad \text{Find } r_{vir}\,, && (13)\\
\nu(M^*) &= 1 \quad \rightarrow \quad \text{Find } M^*\,, && (14)\\
c(M_{vir},z) &= \frac{9}{1+z}\left[\frac{M_{vir}}{M^*}\right]^{-0.13} \quad (\text{Bullock et al. 2001}) \quad \rightarrow \quad \text{Find } c\,, && (15)\\
M_{vir} &= 4\pi\rho_s\frac{r_{vir}^3}{c^3}\left[\ln(1+c)-\frac{c}{1+c}\right] \quad \rightarrow \quad \text{Find } \rho_s\,, && (16)
\end{aligned}
$$

Finally compute numerically or analytically (see Cooray & Sheth 2002) the Fourier Transform for the spherically symmetric halo profile at $z=0$:

$$
u(k|M_{vir}) = \int_0^{r_{vir}} dr\ 4\pi r^2 \frac{\sin kr}{kr}\frac{\rho(r|M_{vir})}{M_{vir}} \tag{17}
$$

Plot $u(k|M_{vir})$ versus k at $z=0$, for $M_{vir} = 10^{10}, 10^{11}, 10^{12}, 10^{13}, 10^{14}, 10^{15}, 10^{16} M_\odot/h$. Show all cases in the same plot, and use log scale in both axes. Do you find something similar to Fig. 9 in Cooray & Sheth 2002 ?

Make sure $u(k|M_{vir}) \rightarrow 1$ as $k \rightarrow 0$ for all values of M_{vir}.

2) Halo Model: Combine the Halo Model ingredients you found before to compute the Halo-Model power spectrum at $z=0$ with the formulae sequence:

$$
\begin{aligned}
P^{1h}(k) &= \int d\ln M \frac{M^2}{\bar{\rho}_{m0}^2} \frac{dn}{d\ln M} |u(k|M)|^2 & (18) \\
P^{2h}(k) &= \left[\int d\ln M \frac{M}{\bar{\rho}_{m0}} \frac{dn}{d\ln M} b(M) u(k|M) \right]^2 P_L(k) & (19) \\
P(k) &= P^{1h}(k) + P^{2h}(k) & (20)
\end{aligned}
$$

Note that as $k \to 0$, $u(k|M) \to 1$, so in Eq. 19 the term within $[\ ]'s \to 1$ (due to the halo bias normalization condition), and therefore $P^{2h}(k) \to P_L(k)$. However for finite and large k, $P^{2h}(k) \neq P_L(k)$. Plot in the same figure using different line types and colors:

- $P_L(k)$ (linear spectrum),
- $P^{1h}(k)$ (1-halo term),
- $P^{2h}(k)$ (2-halo term),
- $P(k)$ (total halo model spectrum),
- $P^{HF}(k)$ (the non-linear spectrum from `halofit` (Takahashi 2012). See nonlinear options inside `CAMB`).

Use log scale in both axes and set the range $k = [10^{-3}, 10^{1}] h/\mathrm{Mpc}$ (change the value of k_{max} in `CAMB` if necessary). Below is a sample plot of $P_L(k)$ and $P^{HF}(k)$ (first and last in the list above). The idea is for you to fill a plot like this with the other terms.

How does your $P(k)$ compare to $P^{HF}(k)$?

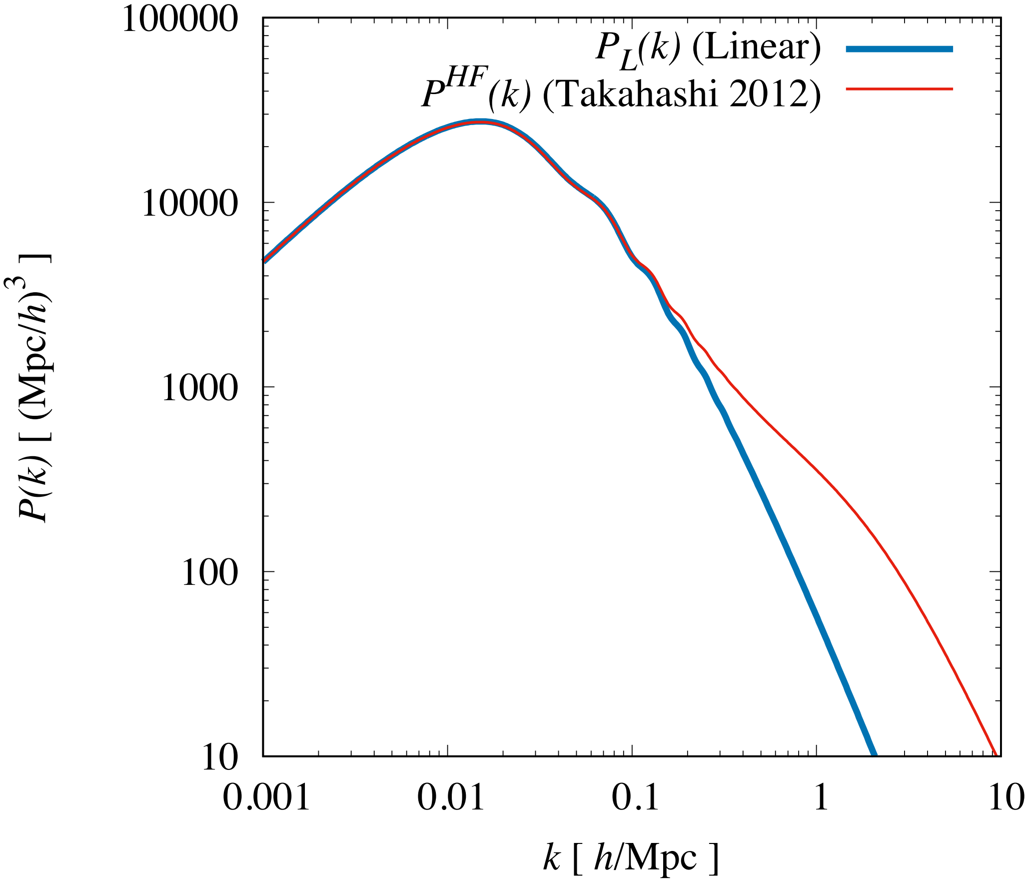

Figure 1: The thick blue line indicates the linear power spectrum $P_L(k)$ from `CAMB`. The thin red line is a fit to simulations for the nonlinear power spectrum $P^{HF}(k)$ from the `halofit` code by Takahashi et al. 2012 (https://arxiv.org/abs/1208.2701). Hopefully your total Halo Model spectrum $P(k)$ will be similar to the `halofit` nonlinear spectrum.