Gauge invariant electrodynamics motivated by a spontaneous breaking of the Lorentz symmetry

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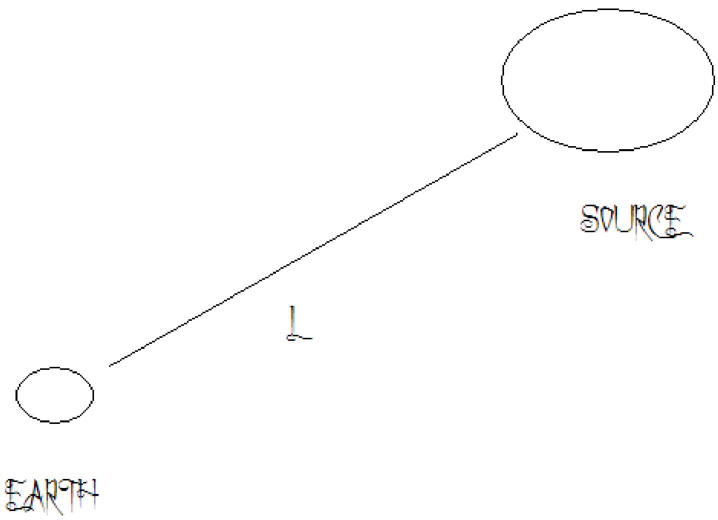
- Motivation
- The model
- Symmetry algebras
- Equations of motion
- Covariant Formulation
- Modified Maxwell Equations
- Insertion in the SME
- Final Comments

Motivation

Gamma Ray Bursts

G. Amelino-Camelia et al., Nature 393(1998)763.



$$L \sim 10^{10} \text{ Light-years}$$

PROPOSITION:

$$\delta v \sim \frac{E}{E_{QG}}$$

produces the structure of GRB with $\Delta t \leq 10^{-3}$ s.
$E_{QG} \sim E_{Planck} = 10^{19}$ Gev.

Dispersion Relation due to Quantum Gravity corrections(Strings, Loop Quantum Gravity, Effective Theories)

J.A., H. Morales-Técotl and L.F. Urrutia, Phys. Rev. Lett. 84(2000)2318.

JA, Phys. Rev.Lett. 94,221302(2005)

Ultra High Energy Cosmic Rays

In this part of the talk we are concerned with the observation of ultra high energy cosmic rays (UHECR), i.e. those cosmic rays with energies greater than $\sim 4 \times 10^{18}$ eV.

- Although not completely clear, it has been suggested that these high energy particles are possibly heavy nuclei (we will assume here that they are protons).

- By virtue of the isotropic distribution with which they arrive to us, they originate in extragalactic sources.

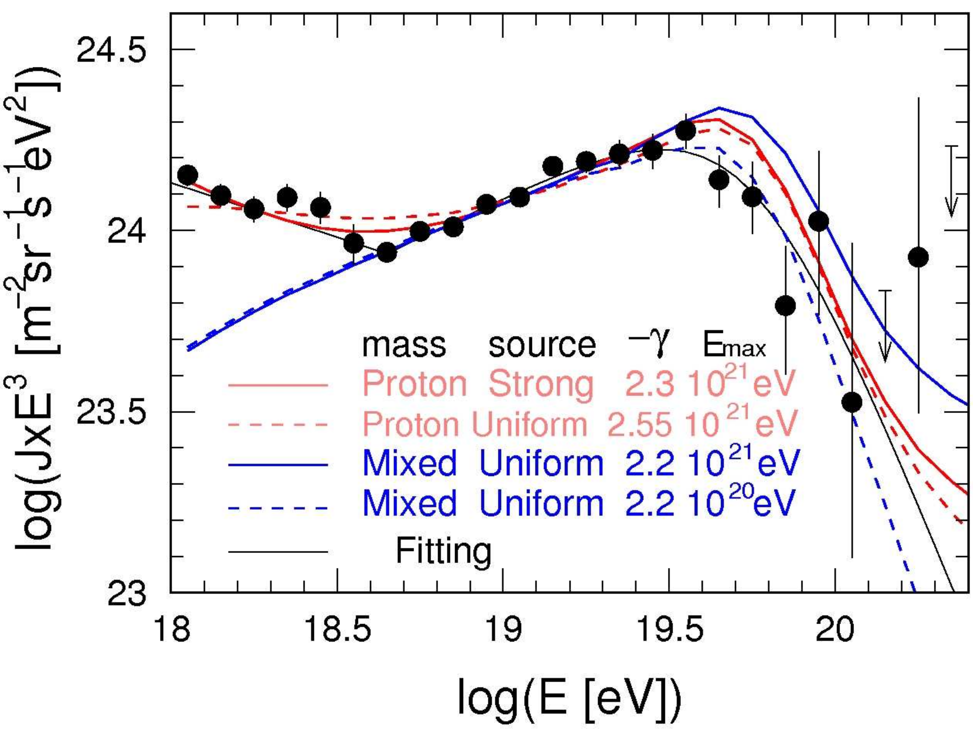
The Greisen-Zatsepin-Kuz'min (GZK) cutoff

-Their propagation in open space is affected by the cosmic microwave background radiation (CMBR), producing a friction on UHECR making them release energy in the form of secondary particles and affecting their possibility to reach great distances.

- Cosmic rays with energies above 1×10^{20} eV should not travel more than ~ 100 Mpc.

The Auger Observatory has recently reported his observations on the highest energy cosmic rays.

They see the GZK cutoff in the flux. But still some of the cosmic rays have a trans GZK energy. This means that Lorentz invariance violation may be necessary to explain their presence, if nearby sources of such cosmic rays are not found.



The combined energy spectrum multiplied by E^3, and the predictions of three astrophysical models. The input assumptions of the models (mass composition at the sources, the source distribution, spectral index and exponential cutoff energy per charge at the acceleration site) are indicated in the figure.

The model

Let us consider the Lagrangian

$$L(F_{\alpha\beta},\ X_{\mu}) = -\ V(F_{\alpha\beta})\ -\ \bar{F}^{\nu\mu}\partial_{\nu}X_{\mu}, \qquad F_{\alpha\beta} = -F_{\beta\alpha} \tag{1}$$

where the dual field $\bar{F}^{\nu\mu}$ is

$$\bar{F}^{\nu\mu} = \frac{1}{2}\epsilon^{\nu\mu\alpha\beta}F_{\alpha\beta}, \tag{2}$$

and the X_{μ} are just Lagrange multipliers that will finally impose the condition that the remaining field strength satisfies the Bianchi identity.
Our conventions are $\eta_{\mu\nu} = diag(+,-,-,-)$, $\epsilon^{0123} = +1$, $\epsilon_{123} = +1$. To introduce a symmetry breaking we proceed in the standard way by considering the potential

$$V(F_{\mu\nu}) = \frac{1}{2}\alpha F^2 + \frac{\beta}{4}\left(F^2\right)^2, \beta > 0. \tag{3}$$

To find the vacuum configuration we have to extremize the effective action, subjected to the condition that $F_{\alpha\beta}$ and X_{α} are constant fields. Applying these requirements to (1) plus the choice (3) we obtain

$$\frac{\partial V}{\partial F^{\mu\nu}} = 0 = \left(\alpha + \beta F^2\right)F_{\mu\nu}, \tag{4}$$

which is solved by a constant $(F_{Extr})_{\alpha\beta} \equiv C_{\alpha\beta}$ subjected to the condition

$$\left(F^2\right)_E = -\frac{\alpha}{\beta} = C^2 \neq 0. \tag{5}$$

The expansion around the minimum $(C_{\mu\nu},\ \ C_\mu)$ is such that

$$
\begin{aligned}
F_{\alpha\beta}(x) &= C_{\alpha\beta}+a_{\alpha\beta}(x) && (6)\\
X_\mu &= C_\mu+\bar{X}_\mu && (7)
\end{aligned}
$$

Next we consider the equations of motion arising from (11) and show that the Lagrange multiplier $\bar{X}_\mu$ is fully determined up to gauge transformation $\bar{X}_\mu \longrightarrow \bar{X}_\mu+\partial_\mu\chi$. The equations are

$$
\begin{aligned}
\delta a_{\alpha\beta} &: \quad -\epsilon^{\nu\mu\alpha\beta}\partial_\nu\bar{X}_\mu-2\frac{\partial V}{\partial a_{\alpha\beta}}=0, && (8)\\
\delta\bar{X}_\mu &: \quad \epsilon^{\nu\mu\alpha\beta}\partial_\nu a_{\alpha\beta}=0. && (9)
\end{aligned}
$$

Eq. (9) establishes that the two form a is closed, i.e. $d\,a=0$. The Hodge-De Rham theorem says that the most general solution to this is obtained by requiring

$$a=d\,A+l\,s, \tag{10}$$

where A is a one form, l is a constant and s is an harmonic two-form. Since in our case we naturally have one such form at our disposal, arising precisely from the chosen vacuum, we take

$$s=\frac{1}{2}C_{\mu\nu}d\,x^\mu d\,x^\nu$$

which is certainly harmonic because it is constant. Calling $d\,A=f$ we have

$$a_{\alpha\beta}(x)=l\,C_{\alpha\beta}+f_{\alpha\beta}(x). \tag{11}$$

It is convenient to rename the dimensionless parameter $l = \xi - 1$. Defining

$$C_{\mu\nu} = \frac{1}{2\xi} D_{\mu\nu}, \qquad B = \frac{\beta}{4} > 0, \tag{12}$$

where

$$\beta = \frac{1}{2(\xi^2 - 1)\big(C^{\alpha\beta} C_{\alpha\beta}\big)}, \qquad \big(\xi^2 - 1\big)\big(C^{\alpha\beta} C_{\alpha\beta}\big) > 0, \tag{13}$$

we obtain our final action

$$S(A_\alpha) = \int d^4x \bigg(- \frac{1}{4}[1 - D^2 B] D^2 - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - B[(D_{\mu\nu} f^{\mu\nu}) + (f_{\mu\nu} f^{\mu\nu})]^2 \bigg) \tag{14}$$

where $f_{\alpha\beta}(x) = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. We recognize the standard Maxwell kinetic term in the RHS of Eq.(14). The only restriction now is $B > 0$, with D^2 arbitrary.

Symmetry algebras arising from different choices of the vacuum

VSR:A. G. Cohen and S. L. Glashow, Very special relativity, Phys. Rev. Lett. 97 (2006) 021601

In this section we study the possible vacua allowed by the tensor symmetry breaking and also identify the corresponding subgroups of the Lorentz group which are left invariant after the breaking. In order to do this, it is convenient to parameterize the background field $D_{\mu\nu}$ in terms of three dimensional components

$$D_{ij} = -\epsilon_{ijm} b_m, \qquad D_{0i} = e_i, \quad \epsilon_{123} = 1 \tag{15}$$

$$[D_{\mu\nu}] = \begin{bmatrix} 0 & e_1 & e_2 & e_3 \\ -e_1 & 0 & -b_3 & b_2 \\ -e_2 & b_3 & 0 & -b_1 \\ -e_3 & -b_2 & b_1 & 0 \end{bmatrix} \tag{16}$$

which will mix when going to another reference frame via a passive Lorentz transformation. Since we have two vectors that determine a plane we choose a coordinate frame where

$$\mathbf{b} = (0,\ 0,\ b), \qquad \mathbf{e} = (0, \quad e_2 = |\mathbf{e}| \sin\ \psi, \quad e_3 = |\mathbf{e}| \cos \psi), \tag{17}$$

That is to say, we have chosen the plane of the two vectors as the $(z - y)$ plane, with the vector **b** defining the z-direction and ψ being the angle between **b** and **e**. In this way the matrix representing the vacuum is

$$[D_{\mu\nu}] = \begin{bmatrix} 0 & 0 & e_2 & e_3 \\ 0 & 0 & -b & 0 \\ -e_2 & b & 0 & 0 \\ -e_3 & 0 & 0 & 0 \end{bmatrix}. \tag{18}$$

The most general infinitesimal generator G, including Lorentz transformations plus dilatations is

$$G = [G^{\mu}{}_{\nu}] = -i \begin{bmatrix} z & x_1 & x_2 & x_3 \\ x_1 & z & -y_3 & y_2 \\ x_2 & y_3 & z & -y_1 \\ x_3 & -y_2 & y_1 & z \end{bmatrix}. \tag{19}$$

Motivated by previous work, we are including conformal dilatations D among our generators. Within this restricted Poincare algebra, this generator commutes with the remaining ones corresponding to pure Lorentz transformations and can be realized as a multiple of the identity. We do this in order to explore the possibility of having the largest possible invariant sub-algebra after the symmetry breaking.

Summarizing, all the two-parameter subalgebras that leave the vacuum invariant are isomorphic to $T(2)$, while the only three-parameter subalgebra , corresponding to the case (E-2), is isomorphic to HOM(2).

Subcase E-2

Here we have

$$b^2 - e_2^2 = 0 \rightarrow b = s\, e_2, \quad s = \pm 1 \tag{20}$$

which corresponds to a plane wave VEV.
We have

$$y_1 = 0, \qquad x_1 = -\, 2s\, z\,, \quad x_2 = -\, s\, y_3,\ y_2 = s\, x_3 \tag{21}$$

Here we are left with a three parameter Lie algebra $(z,\ x_3,\ y_3)$ and the generator is

$$G = -\, z\big(2s\ K^1 + i\ I\big) + x_3\big(K^3 - s\ J^2\big) - y_3\big(s\ K^2 + J^3\big) \tag{22}$$

Defining

$$G_z = -\,\big(2s\, K^1 + i\, I\big), \quad G_x = K^3 - s\, J^2,\ \ G_y = -\,\big(J^3 + s\, K^2\big) \tag{23}$$

we obtain the algebra

$$[G_z,\ G_x] = 2i\ G_x, \quad [G_z,\ G_y] = 2i\ G_y, \qquad [G_x,\ G_y] = 0 \tag{24}$$

which is isomorphic to HOM(2).

The equations of motion: the propagating sector

To study the propagation properties we consider only the quadratic terms in the Lagrangian

$$L_0 = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - B(f_{\mu\nu} D^{\mu\nu})^2, \tag{25}$$

where we recall that $f_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$. The equations are

$$\left(\partial^2 A_\beta - \partial_\beta \partial^\alpha A_\alpha\right) = -8B\, D_{\alpha\beta} \partial^\alpha (D^{\mu\nu} \partial_\mu A_\nu) \tag{26}$$

We have verified the consistency of the above when taking ∂^β.

It is convenient to define

$$X = D^{\mu\nu} \partial_\mu A_\nu \tag{27}$$

and to introduce the notation

$$D_{\alpha k} \partial^\alpha = D_k = [D_{0k} \partial_0 - D_{lk} \partial_l] \tag{28}$$

$$D_0 = D_{i0} \partial_i \tag{29}$$

In this way we have

$$X = -(D_j A_j + D_0 A_0), \tag{30}$$

together with

$$D_0 \partial_0 = -\partial_l D_l. \tag{31}$$

Covariant formulation

The modified dispersion relations are found very easily by manipulating Eq.(26). In momentum space

$$A_\mu(x) = \int d^4x \, A_\mu(k) e^{-ik_\mu x^\mu} \tag{32}$$

and choosing the Lorentz gauge, these equations reduce to

$$k^2 A_\mu + 2p_\mu(p^\nu A_\nu) = 0 \quad , \quad p^\alpha \equiv 2\sqrt{B} D^{\alpha\beta} k_\beta \tag{33}$$

$$k^\nu A_\nu = 0 \quad . \tag{34}$$

The vector p^α is proportional to the momentum space version of the vector D_μ introducced in Eqs. (28) and (29).
Multiplying the first relation in (33) by p^μ, it follows that:

$$(k^2 + 2p^2)(p^\nu A_\nu) = 0. \tag{35}$$

Moreover $p^\nu A_\nu$ is gauge invariant. In fact, in coordinate space is proportional to $D^{\alpha\beta} f_{\alpha\beta}$. So this component is physical and has dispersion relation given by:

$$k^2 + 2p^2 = 0. \tag{36}$$

If $(k^2 + 2p^2)$ is not zero in Eq.(35), it follows that $p^\nu A_\nu = 0$. In four dimensions, this condition plus the Lorentz gauge leaves two degrees of freedom. But the Lorentz gauge permits a further gauge transformation with parameter λ such that:

$$\partial^2 \lambda = 0, \tag{37}$$

which leaves only one degree of freedom as it should be. Moreover from the first equation in (33), this remaining degree of freedom satisfies

$$k^2 A_\mu = 0, \tag{38}$$

so its dispersion relation is

$$k^2 = 0. \tag{39}$$

The general solution of (35) is:

$$p^\nu A_\nu = -\lambda_1(k)\delta(k^2 + 2p^2). \tag{40}$$

Putting it back into (33), we get

$$A_\mu = a_\mu(k)\delta(k^2) + 2\frac{\lambda_1(k)}{k^2}\delta(k^2 + 2p^2)p_\mu, \quad a_\mu(k)k^\mu = 0, a_\mu(k)p^\mu = 0 \tag{41}$$

From (41), we get the electromagnetic tensor

$$f_{\mu\nu} = (k_\mu a_\nu(k) - k_\nu a_\mu(k))\delta(k^2) + 2\frac{\lambda_1(k)}{k^2}\delta(k^2 + 2p^2)(k_\mu p_\nu - k_\nu p_\mu) \tag{42}$$

It represents a plane wave with dispersion relation $k^2 = 0$ (The magnetic and electric fields are the normal ones, perpendicular to $\mathbf{k}$), plus a plane wave propagating in the direction $\mathbf{k}$ with dispersion relation $k^2 + 2p^2 = 0$.

The fields for the second wave are

$$E_j = f_{0j} = 2(k_0 p_j - k_j p_0)\frac{\lambda_1(k)}{k^2} \qquad B_j = 2\epsilon_{jkl}(k_k p_l)\frac{\lambda_1(k)}{k^2} \tag{43}$$

Notice that $\mathbf{E}$ is perpendicular to $\mathbf{B}$, $\mathbf{B}$ is perpendicular to $\mathbf{k}$, but $\mathbf{E}$ is not necessarily orthogonal to $\mathbf{k}$. Moreover, this wave exists only if $k^2 \neq 0$.

Modified Maxwell equations

$$\begin{aligned} \nabla \cdot \mathbf{D} &= 4\pi\rho, \quad \nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c}\mathbf{J}, &(44)\\ \nabla \cdot \mathbf{B} &= 0, \quad \nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0. &(45)\end{aligned}$$

In components

$$\begin{aligned} D_i &= (\delta_{ij} - 8B\, e_i e_j)E_j + 8B\, e_i b_j B_j, &(46)\\ H_i &= (\delta_{ij} + 8b\, b_i b_j)B_j - 8B\, b_i e_j E_j. &(47)\end{aligned}$$

Summary of the dispersion relations and electromagnetic fields

We assume the space-time dependence of any field to be proportional to $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$, where $\mathbf{k}$ is the momentum of the wave and we work with Coulomb gauge potential $\mathbf{A}$. The notation is

$$\mathbf{b} = \{b_m\}, \quad \mathbf{e} = \{e_m\}\,. \tag{48}$$

The expressions of Eqs. (28) , (29) in momentum space are

$$\begin{aligned} \{D_k\} &= \mathbf{D} = -i[\omega\mathbf{e} - \mathbf{k}\times\mathbf{b}] &(49)\\ D_0 &= -i\mathbf{e}\cdot\mathbf{k} &(50)\end{aligned}$$

There are two main cases:

(i) $D_j A_j = 0$: In this case the dispersion relation is $\omega = |\mathbf{k}|$. The fields are

$$\mathbf{B} = \gamma\{(\mathbf{e}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}} - \mathbf{e} + \mathbf{b}\times\hat{\mathbf{k}}\} \quad , \tag{51}$$

$$\mathbf{E} = \gamma\{(\mathbf{b}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}} - \mathbf{b} - \mathbf{e}\times\hat{\mathbf{k}}\} \quad . \tag{52}$$

(ii) $D_j A_j \neq 0$: In this case the dispersion relation and the fields are

$$\omega = |\mathbf{k}|\frac{\sqrt{256B^2(\hat{\mathbf{k}}\cdot(\mathbf{b}\times\mathbf{e}))^2 - 4(1-8B\mathbf{e}^2)\{8B[(\hat{\mathbf{k}}\cdot\mathbf{e})^2 - (\hat{\mathbf{k}}\times\mathbf{b})^2] - 1\}} - 16B\hat{\mathbf{k}}\cdot(\mathbf{b}\times\mathbf{e})}{2(1-8B\mathbf{e}^2)}, \tag{53}$$

$$\mathbf{B} = \gamma\{\mathbf{k}\times(\mathbf{k}\times\mathbf{b}) - \omega\mathbf{k}\times\mathbf{e}\} \quad , \tag{54}$$

$$\mathbf{E} = \gamma\{\omega\mathbf{k}\times\mathbf{b} - \omega^2\mathbf{e} + \mathbf{k}(\mathbf{e}\cdot\mathbf{k})\} \quad . \tag{55}$$

In both cases γ is an arbitrary constant. For small B, the dispersion relation (53) reduces to

$$\omega = |\mathbf{k}|\Big[1 + B\Big(8\hat{\mathbf{k}}\cdot(\mathbf{e}\times\mathbf{b}) + 4(\hat{\mathbf{k}}\times\mathbf{b})^2 + 4(\hat{\mathbf{k}}\times\mathbf{e})^2\Big)\Big]. \tag{56}$$

The anisotropic velocity of light arising from the above dispersion relation is

$$\nabla_{\mathbf{k}}\omega = \mathbf{c}(\hat{\mathbf{k}}) = \hat{\mathbf{k}}\Big(1 + 8B\big(b^2+e^2\big) - 4B\Big((\hat{\mathbf{k}}\times\mathbf{b})^2 + (\hat{\mathbf{k}}\times\mathbf{e})^2\Big)\Big) + 8B(\mathbf{e}\times\mathbf{b}) - 8B\Big(\mathbf{b}\cdot\hat{\mathbf{k}}\Big)\mathbf{b} - 8B\Big(\mathbf{e}\cdot\hat{\mathbf{k}}\Big)\mathbf{e}. \tag{57}$$

THE MODEL AS A SECTOR OF THE SME

In order to impose some bounds upon the parameters of the model it is convenient to recast the quadratic sector of the action (14) in the language of the Standard Model Extension and to make use of the numerous experimental constraints derived from it. To begin with, let us recall the dimension of the fields and parameters involved in the model

$$[A] = m, \qquad [e] = [b] = m^2, \qquad [B] = \frac{1}{m^4}, \qquad [B\, e^2] = 0 \tag{58}$$

Next we make the identification

$$-B(f_{\mu\nu}D^{\mu\nu})^2 = -\frac{1}{4}(k_F)^{\kappa\lambda\mu\nu} f_{\kappa\lambda} f_{\mu\nu}, \tag{59}$$

where the tensor $(k_F)^{\kappa\lambda\mu\nu}$, with 19 independent components, has all the symmetries of the Riemann tensor and a vanishing double trace. In terms of the matrix elements $D_{\mu\nu}$ characterizing the vacuum expectation values of the electromagnetic tensor we obtain

$$(k_F)^{\kappa\lambda\mu\nu} = 4B\left[D^{\kappa\lambda}D^{\mu\nu} + \frac{1}{2}\left(D^{\kappa\mu}D^{\lambda\nu} - D^{\lambda\mu}D^{\kappa\nu}\right)\right] - \frac{1}{2}B\, D^2\left(\eta^{\kappa\mu}\eta^{\lambda\nu} - \eta^{\lambda\mu}\eta^{\kappa\nu}\right), \tag{60}$$

where

$$D^2 \equiv D_{\alpha\beta}D^{\alpha\beta} = 2(\mathbf{b}^2 - \mathbf{e}^2). \tag{61}$$

Next we have to identify the appropriate combinations of the components of $(k_F)^{\kappa\lambda\mu\nu}$ which are bounded.

The finally required combinations, being dimensionless numbers, are

$$(\bar{\kappa}_{e+})^{jk} = \frac{1}{2}(\kappa_{DE} + \kappa_{HB})^{jk} = 6B \left[b_j b_k - e_j e_k - \frac{1}{3}\left(\mathbf{b}^2 - \mathbf{e}^2\right)\delta_{jk}\right] < 10^{-32}, \tag{62}$$

$$\begin{aligned}(\bar{\kappa}_{e-})^{jk} &= \frac{1}{2}(\kappa_{DE} - \kappa_{HB})^{jk} - \frac{1}{3}\delta_{jk} tr(\kappa_{DE}) \\ &= -6B\left[(b_j b_k + e_j e_k) - \frac{1}{3}\delta_{jk}(\mathbf{b}^2 + \mathbf{e}^2)\right] < 10^{-16},\end{aligned} \tag{63}$$

$$(\bar{\kappa}_{o+})^{jk} = \frac{1}{2}(\kappa_{DB} + \kappa_{HE})^{jk} = 6B(e_j b_k - e_k b_j) < 10^{-12}, \tag{64}$$

$$(\bar{\kappa}_{o-})^{jk} = \frac{1}{2}(\kappa_{DB} - \kappa_{HE})^{jk} = 6B\left[(e_j b_k + e_k b_j) - \frac{2}{3}(\mathbf{e}\cdot\mathbf{b})\delta_{jk}\right] < 10^{-32}, \tag{65}$$

$$\bar{\kappa}_{tr} = \frac{1}{3} t\ r(k_{DE})^{j} = -2B(\mathbf{e}^2 + \mathbf{b}^2) < 10^{-15}. \tag{66}$$

All matrices from (62) to (65) are traceless, with $(\bar{\kappa}_{0+})^{jk}$ being antisymmetric (3 independent components) while the three remaining ones are symmetric (5 independent components each). The above combinations (62) to (66) constitute a convenient alternative way to display the 19 independent components

of the tensor $(k_F)^{\alpha\beta\mu\nu}$. The bounds are obtained in terms of such combinations referred to the Standard Inertial Reference Frame centered in the Sun.
Notice that the bilinears which are odd under the duality transformations

$$\mathbf{e} \rightarrow \mathbf{b}, \quad \mathbf{b} \rightarrow -\mathbf{e}. \tag{67}$$

are much more constrained than those which are even. In this way, even though our model is not duality invariant, this transformation seems to explain the above mentioned hierarchy in the LIV parameters exhibited in Eqs. (62)-(66).
In order to obtain more specific qualitative consequences of the bounds (62) to (66) it is convenient to express them in the coordinate system defined by (17). Also we introduce the notation

$$\mathbf{x} = \sqrt{6B}\mathbf{e} \times 10^{16}, \quad \mathbf{y} = \sqrt{6B}\mathbf{b} \times 10^{16}, \qquad x = |\mathbf{x}|, \\ y = |\mathbf{y}| \tag{68}$$

We consider only absolute values of the related quantities and we focus upon those constraints arising from Eqs. (62) and (65) . The non-trivial contributions are

$$\left|(\tilde{\kappa}_{e+})^{11}\right| = \left|(x^2 - y^2)\right| < 3 \tag{69}$$

$$\left|(\tilde{\kappa}_{e+})^{22}\right| = \left|\left(1 - 3\sin^2\psi\right)x^2 - y^2\right| < 3 \tag{70}$$

$$\left|(\tilde{\kappa}_{e+})^{33}\right| = \left|2y^2 + \left(1 - 3\cos^2\psi\right)x^2\right| < 3 \tag{71}$$

$$\left|(\tilde{\kappa}_{e+})^{23}\right| = \left|x^2 \sin 2\psi\right| < 2 \tag{72}$$

together with

$$(\tilde{\kappa}_{o\,-})^{11} \;=\; (\tilde{\kappa}_{o\,-})^{22} = |x\,y\cos\psi| < \frac{3}{2} \tag{73}$$

$$(\tilde{\kappa}_{o\,-})^{33} \;=\; |x\,y\cos\psi| < \frac{3}{4} \tag{74}$$

$$(\tilde{\kappa}_{o\,-})^{23} \;=\; |x\,y\sin\psi| < 1 \tag{75}$$

The allowed region in the $(x - y)$ plane is shown in Fig. 1 where we plot the boundaries of the corresponding inequalities. In expressions (72), (73)-(75) we consider the lower bound corresponding to the maximum value of the trigonometric function on the LHS of the inequality. This leads to

$$x < \sqrt{2}, \;\; |x\,y| < \frac{3}{4} < 1 < \frac{3}{2} \tag{76}$$

The boundary $x\,y = 3/4$ is in dashed line. From (71) the most stringent bound arises from $\psi = \pi/2$ and corresponds to

$$y < \sqrt{\frac{3 - x^2}{2}} \tag{77}$$

The boundary is in dot-dot-dashed line. From (70) the most stringent bound arises again from $\psi = \pi/2$ and corresponds to

$$y < \sqrt{3 - 2x^2}, \tag{78}$$

with boundary in dot-dahed line. The expression (69) does not provide additional bounds and is plotted for completeness. The corresponding boundaries are $y_\pm = \sqrt{x^2 \pm 3}$, shown in dotted lines in Fig.

1. An upper bound including all previous ones is given by the interior of the circle

$$y = \sqrt{\frac{3}{2} - x^2}, \tag{79}$$

shown in solid line, and which yields the bound

$$B(\mathbf{e}^2 + \mathbf{b}^2) = \frac{1}{M_B^4}(\mathbf{e}^2 + \mathbf{b}^2) < 2.5 \times 10^{-33}. \tag{80}$$

This bound includes those of Eq. (62) to Eq. (66). All the above relations are valid in the Standard Inertial Reference Frame centered in the Sun.

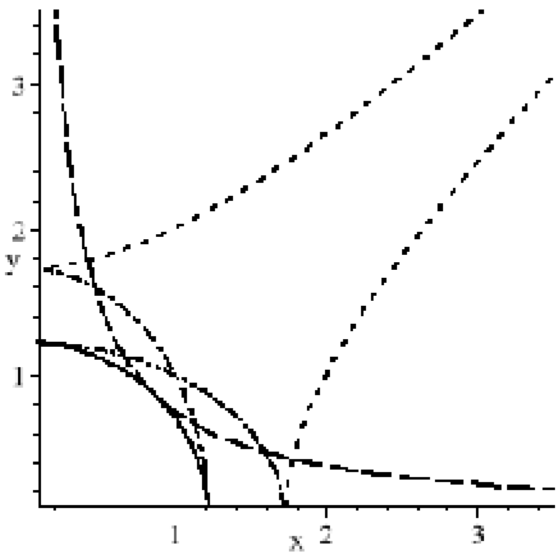


Figure 1. Boundaries of the allowed region obtained from the constraints in the parameters $\tilde{\kappa}_{e+}^{jk}$ and $\tilde{\kappa}_{o-}^{jk}$. The allowed region is on the inside the dashed, dot-dashed and dot-dot-dashed lines. An excellent approximation for it is the circle shown in solid line.

SUMMARY

– We have studied a novel way of implementing a model with spontaneously broken Lorentz symmetry by introducing a constant vacuum expectation value (VEV) of the field strength $\langle F_{\mu\nu}\rangle = C_{\mu\nu}$, so our model preserves gauge invariance from the very beginning.

– There is only one case, given in the subsubsection E-2, which results in the breaking of the Lorentz group to the three generators subgroup $HOM(2)$. All the other cases break to a subgroup isomorphic to $T(2)$, with two generators.

– Anisotropy in the speed of light:
The two-way light speed is defined by:

$$c_{TW}(\hat{\mathbf{k}}) = \frac{1}{2}\Big(c(\hat{\mathbf{k}}) + c(-\hat{\mathbf{k}})\Big) = 1 + 8B\big(e^2 + b^2\big) - 4B\Big(\big(\mathbf{b}\cdot\hat{\mathbf{k}}\big)^2 + \big(\mathbf{e}\cdot\hat{\mathbf{k}}\big)^2\Big) \qquad (81)$$

An appropriate definition in this model of the anisotropy of the speed of light is

$$\frac{\Delta c}{c} \equiv \Big|c_{TW}(\hat{\mathbf{k}}) - c_{TW}(\hat{\mathbf{k}}\times\big(\hat{\mathbf{k}}\times\hat{\mathbf{n}}\big))\Big|, \qquad (82)$$

$$\frac{\Delta c}{c} = \Big|4B(1-\big(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}\big)^2)\Big(b^2\big(\hat{\mathbf{b}}\cdot\hat{\mathbf{k}}\big)^2 + e^2\big(\hat{\mathbf{e}}\cdot\hat{\mathbf{k}}\big)^2\Big)\Big|. \qquad (83)$$

From the last expression we obtain the bound

$$\frac{\Delta c}{c} < 4B \sin{}^2\theta(\mathbf{b}^2 + \mathbf{e}^2) < 4B(\mathbf{b}^2 + \mathbf{e}^2) < 10^{-32}, \tag{84}$$

according to (80), where θ is the angle between the vector $\hat{\mathbf{n}}$ and $\hat{\mathbf{k}}$. The above anisotropy measures the difference in the two-speed of light propagating in perpendicular directions in a given reference frame.

– Assuming our background fields $\mathbf{e}$ and $\mathbf{b}$ might represent some galactic or intergalactic fields in the actual era, we obtain a very reasonable bound for the magnetic intergalactic field by assuming that the constant appearing in the action (14) corresponds to an energy density ρ

$$\rho = \frac{1}{4}\left[(1 - D^2 B\right] D^2 \simeq \frac{1}{2}\left(\mathbf{b}^2 - \mathbf{e}^2\right), \tag{85}$$

which we can associate to the cosmological constant, since it would represent a global property of the universe. The fact that this constant is positive favors $\left(\mathbf{b}^2 - \mathbf{e}^2\right) > 0$, so that one can perform a passive Lorentz transformation to a reference frame where $\mathbf{e} = \mathbf{0}$. Suppossing further that this frame, which describes the intergalactic fields, is concordant with the Standard Inertial Reference Frame, in such a way that the bounds change by small amounts and taking the upper limit

$$|\rho_\Lambda| < 10^{-48}\,(G\,e\,V)^4, \tag{86}$$

we obtain the bound

$$|\mathbf{b}| \; < \; 5 \times 10^{-5}\,G\,a\,u\,s\,s, \tag{87}$$

which is consistent with observations of intergalactic magnetic fields.

THANK YOU!