Branes and CS Gravities

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New Trends in Quantum Gravity

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Background

- 2+1 gravity is a reasonably good model to mimic real (3+1) spacetime dynamics.
- The 2+1black hole has a spectrum labeled by the mass ($M \geq 0$) and the angular momentum (J), with $M \geq |J|$.
- The solutions for $M < |J|$, are naked singularities ("*Green slime and lost socks could emerge from them*").
- NS of different kinds generically appear in numerical collapse experiments (Christodoulou), but they don't necessarily break physical laws.
- What kind of NS are these $M < 0$ 2+1 bhs? Are they dangerous? Are they stable? Can they form?

What we have learned

- $M<0$, $J=0$ states are rather harmless NS: Topological defects $M \Leftrightarrow$ angular deficit; static *0*-branes.
- These states can also have angular momentum; for $M = -|J| > -1$, they are BPS states.
- Similar *0*-branes exist in CS gravity for $D=2n+1$; they are also negative energy states in the bh spectrum.
- *2p*-branes can be similarly constructed; they are natural sources CS for gravity.
- More generally, *2p*-branes couple naturally to CS forms for other nonabelian connections.

- The coupling between *2p*-branes and *2n+1* CS forms circumvents an old obstruction:

A (p-1)-brane source could not couple consistently to a nonabelian p-form. The coupling

$$j^{\mu_1\mu_2\cdots\mu_p} A_{\mu_1\mu_2\cdots\mu_p}$$

Fundamental field

leads to inconsistent time evolution (M.Henneaux and C.Teitelboim, 1986).

- The coupling between nonabelian connections and *2p* branes also provides a mechanism to couple CS theories to matter sources, which can be useful in setting up a perturbative expansion for CS theories in general and for 2+1gravity in particular.

1. Review of the 2+1 black hole

The 2+1 black hole (BBZ)

Static case

$J=0$

$$ds^2 = -(r^2 - M)dt^2 + \frac{dr^2}{(r^2 - M)} + r^2 d\phi^2$$

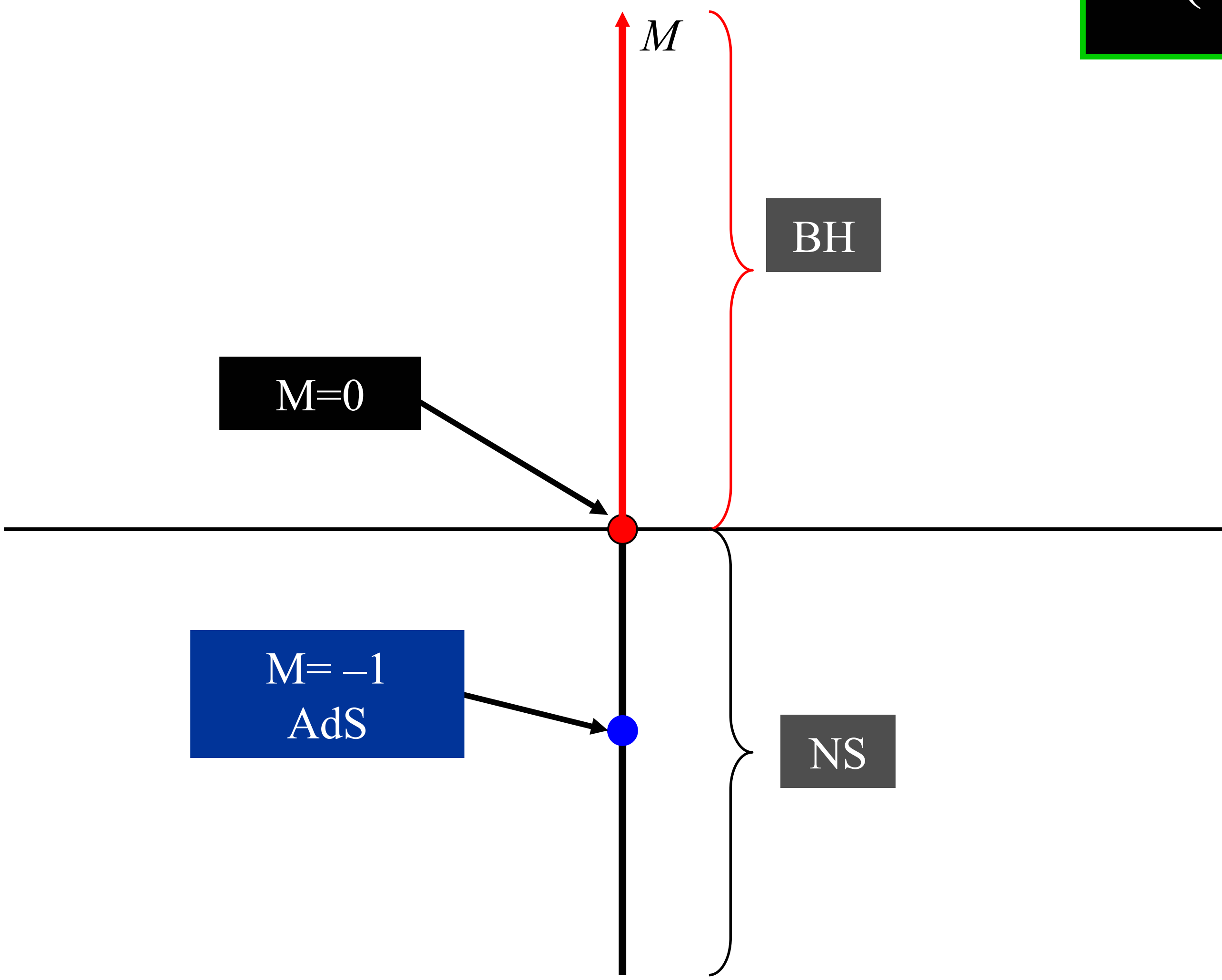
$M \geq 0$ → Black hole; horizon $r_+ = M^{1/2}$

$M = -1$ → AdS spacetime ($\Lambda = -1$)

$M<0$ → No horizon, Naked singularity

2+1 BH spectrum

(J=0)



Spinning 2+1 bh…

$J \neq 0$ case

$$ds^2 = -f^2(r)dt^2 + \frac{dr^2}{f^2(r)} + r^2(Ndt + d\phi)^2$$

$$f^2 = -M + r^2 + \frac{J^2}{4r^2}$$

$$N = -\frac{J}{2r^2}$$

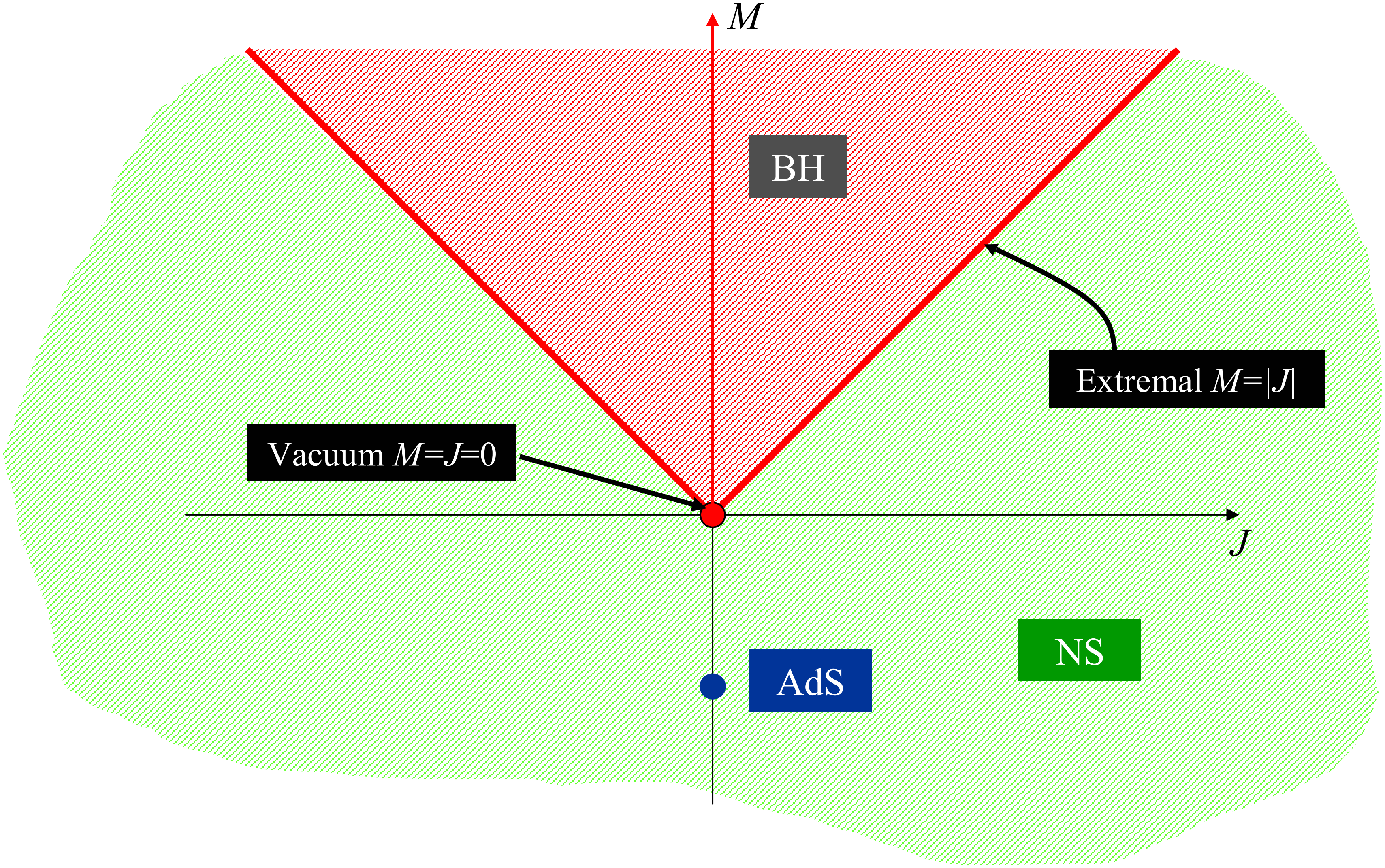
$M \geq |J|$ → BH; horizons $r_\pm \geq 0$

$M = -1, J = 0$ → AdS

$M < |J|, \neq -1$ → Naked singularites

$$r_\pm^2 = \frac{M}{2}\left(1 \pm \sqrt{1 - \frac{J^2}{M^2}}\right) \geq 0, \quad \text{for } M \geq |J|$$

2+1 Black hole spectrum



- Black holes are localized, static, rotationally invariant objects. Their local geometry is AdS_{2+1}

- They are obtained identifying by a Killing vector

$$\xi = \frac{1}{2}\xi^{ab}(x_a\partial_b - x_b\partial_a)$$
$$= \frac{1}{2}\xi^{ab}J_{ab}$$

where AdS_{2+1} is defined by the pseudosphere

$$-(x^0)^2 - (x^1)^2 + (x^2)^2 + (x^3)^2 = -1$$

Black hole identifications

AdS_{2+1}: $-(x^0)^2-(x^1)^2+(x^2)^2+(x^3)^2=-1$

- $\xi_{+-}=r_+J_{12}-r_-J_{03}$ Generic bh, $r_+ > r_- \geq 0$.
- $\xi_{Ext}=r_+(J_{01}-J_{23})+\frac{1}{2}(J_{12}+J_{03}+J_{02}-J_{13})$
  Extremal bh, $r_+ = r_- > 0$.
- $\xi_{Vac}=\frac{1}{2}(J_{12}+J_{03}+J_{02}-J_{13})$
  Vacuum bh, $r_+ = r_- = 0$, or $M=J=0$.

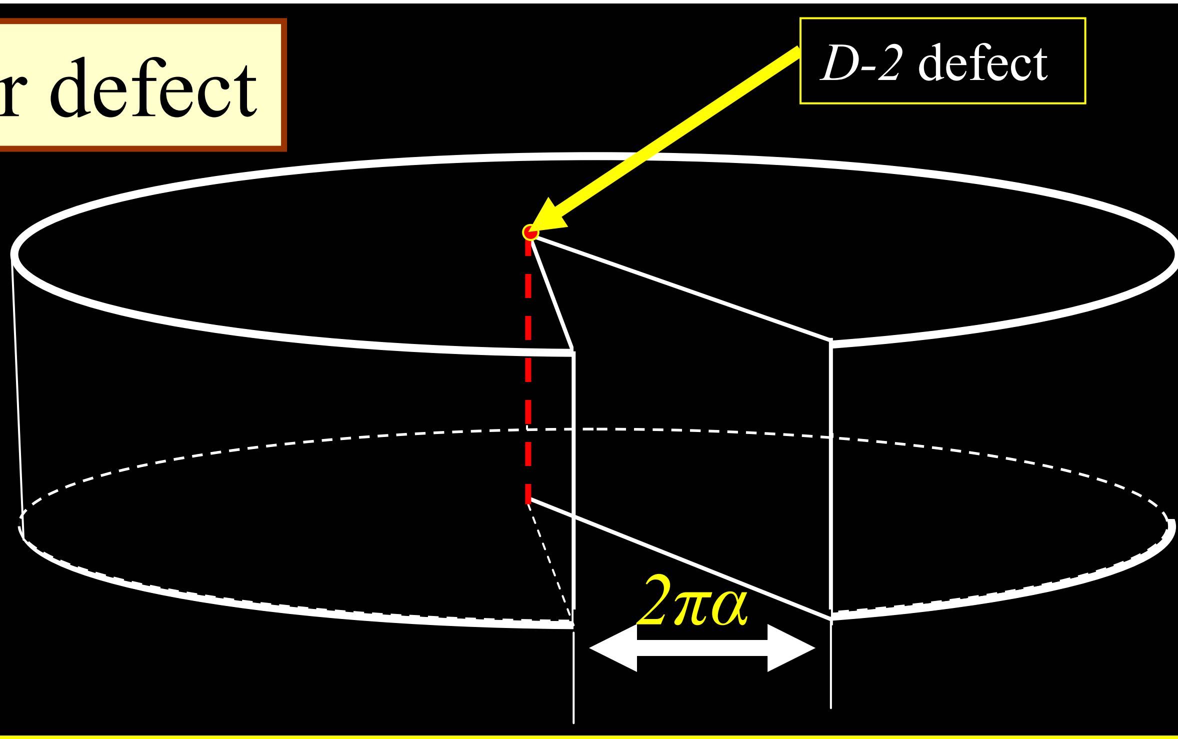
All of these are non compact elements of SO(2,2) (AdS boosts). They leave no fixed points. Hence, no conical singularities and no sources.

2. Angular defects & conical singularities

Topological defects can also be localized, static, rotationally invariant. Their local geometry is AdS_{2+1} as well.

These defects are also produced by identifications with Killing vectors. They belong to the compact part of SO(2,2) (rotations), that have fixed points.

Angular defect

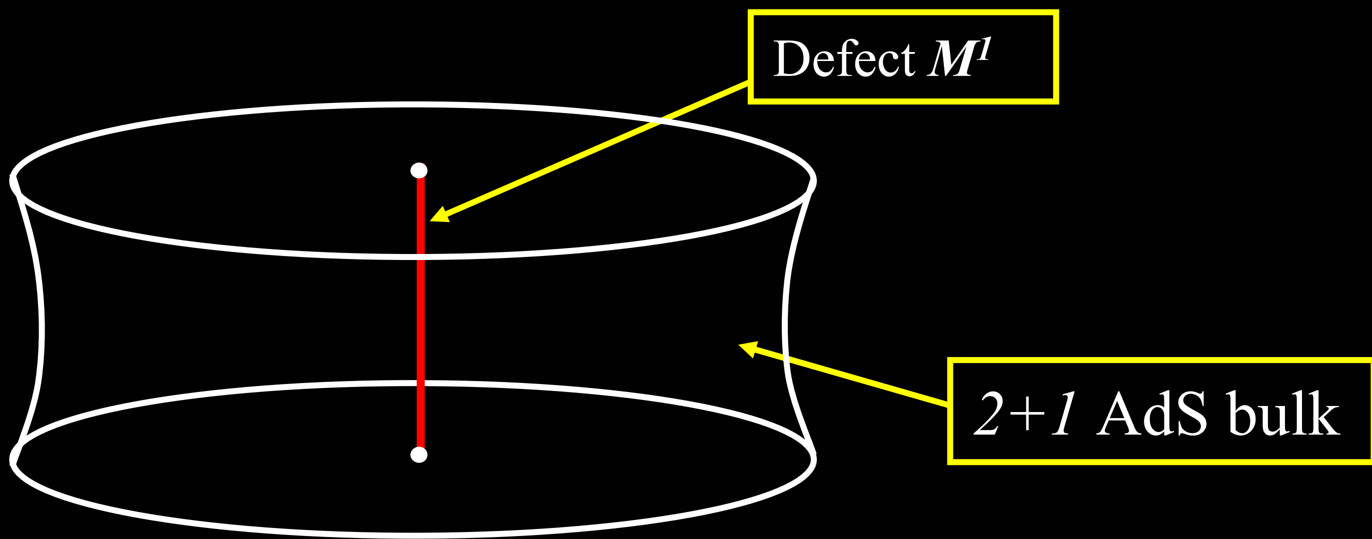


Identification in the x^1-x^2 plane generates a conical singularity at the set of fixed points

Killing vector: $\xi = -2\pi\alpha\,(x^1\partial^2 - x^2\,\partial^1)$
$= -2\pi\alpha\,\partial\varphi$

Angular deficit ⟺ Curvature has δ-singularity

0-brane in 2+1 gravity



The defect appears as a conical singularity in AdS:

$$ds^2 = -\left(\frac{\rho^2}{l^2}+1\right)d\tau^2 + \left(\frac{\rho^2}{l^2}+1\right)^{-1} d\rho^2 + \rho^2 (1-\alpha)^2 d\phi^2$$

and the curvature gains a singularity

Source

$$R^{ab} + \frac{e^a e^b}{l^2} = 2\pi\alpha\delta^{[ab]}_{[12]}\delta^{(2)}(x^1 x^2)dx^1 \wedge dx^2, \quad 0 \leq \alpha \leq 1$$

In appropriate coordinates, this looks like a black hole:

$$ds^2 = -\left(\frac{r^2}{l^2} - M\right)d\tau^2 + \left(\frac{r^2}{l^2} - M\right)^{-1} dr^2 + r^2 d\phi^2$$

But, with negative mass:

$$M = -(1-\alpha)^2$$

The exceptional cases are:

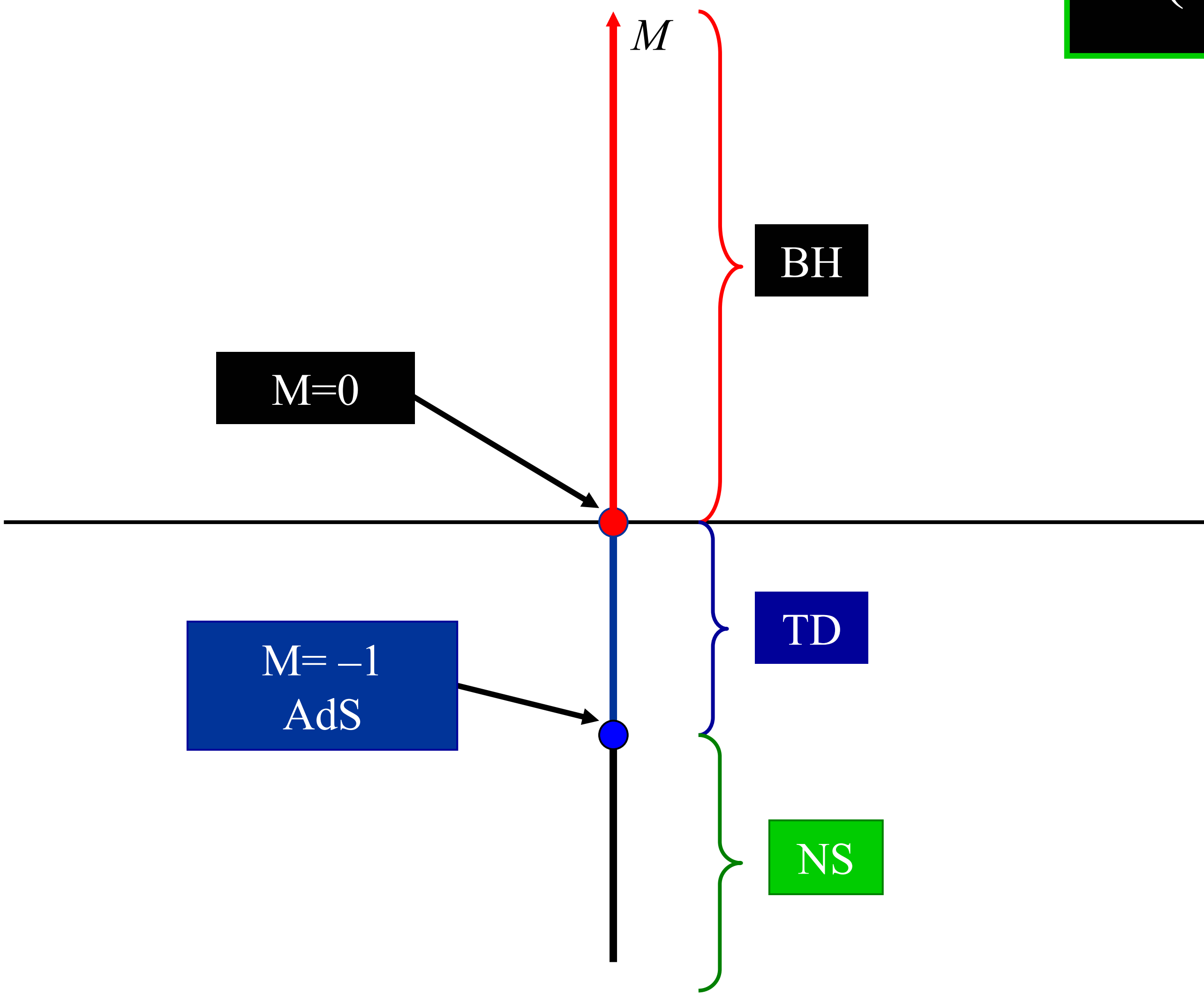
$\alpha = 0, M = -1$ → no defect, AdS spacetime

$\alpha = 1, M = 0$ → maximum defect, vacuum bh

For $-1 < M < 0$, these are ***naked*** but otherwise harmless singularities ***… like all branes***.

2+1 BH spectrum

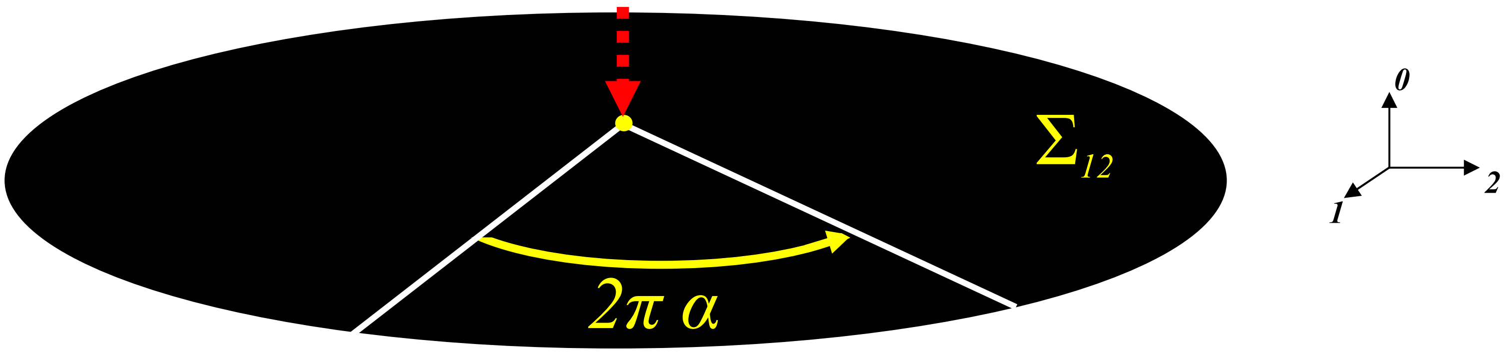
(*J*=0)



The angular defect is produced by identification with a spatial Killing vector with a fixed point: $\xi = -2\pi\,\alpha\,\partial_\varphi$

The length of the Killing vector is proportional to the angular deficit (α) and to the curvature singularity,

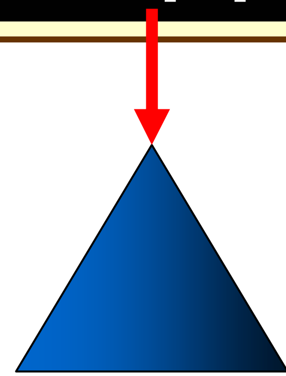
$$R^{ab} + e^a e^b = 2\pi\alpha\delta^{[ab]}_{[12]}\delta(\Sigma_{12})d\Omega^2_\Sigma \qquad T^a = 0$$



Conical singularity

In the case of a flat 2-dim. plane, an identification with a fixed point produces a conical curvature singularity:

$$R^{ab} = 2\pi\alpha\delta^{[ab]}_{[12]}\delta(\Sigma_{12})d\Omega^2_\Sigma$$

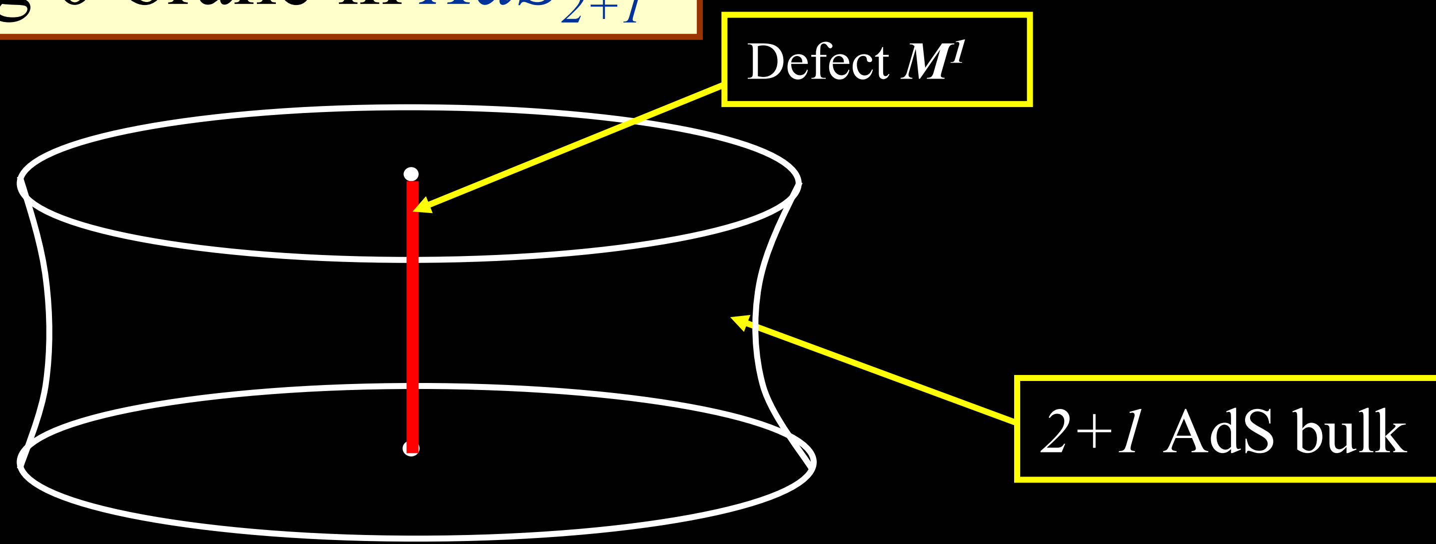


This can also be seen as the introduction of a source (external current)

$$\begin{aligned} F &= j \\ &= 2\pi\alpha\delta(\Sigma_{12})d\Omega^{12}J_{12} \end{aligned}$$

3. Spinning NS in AdS_3 and BPS states

Spinning 0-brane in AdS_{2+1}



Choosing J_{ab} as a linear combination of a spatial rotation and an AdS boost to make the identification, a conical singularity is produced, with $M < 0$ and J:

$$ds^2 = -f^2(r)dt^2 + f^{-2}(r)dr^2 + r^2(Ndt + d\phi)^2$$

$$f^2 = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}, \quad N = -\frac{r_+ r_-}{r^2}.$$

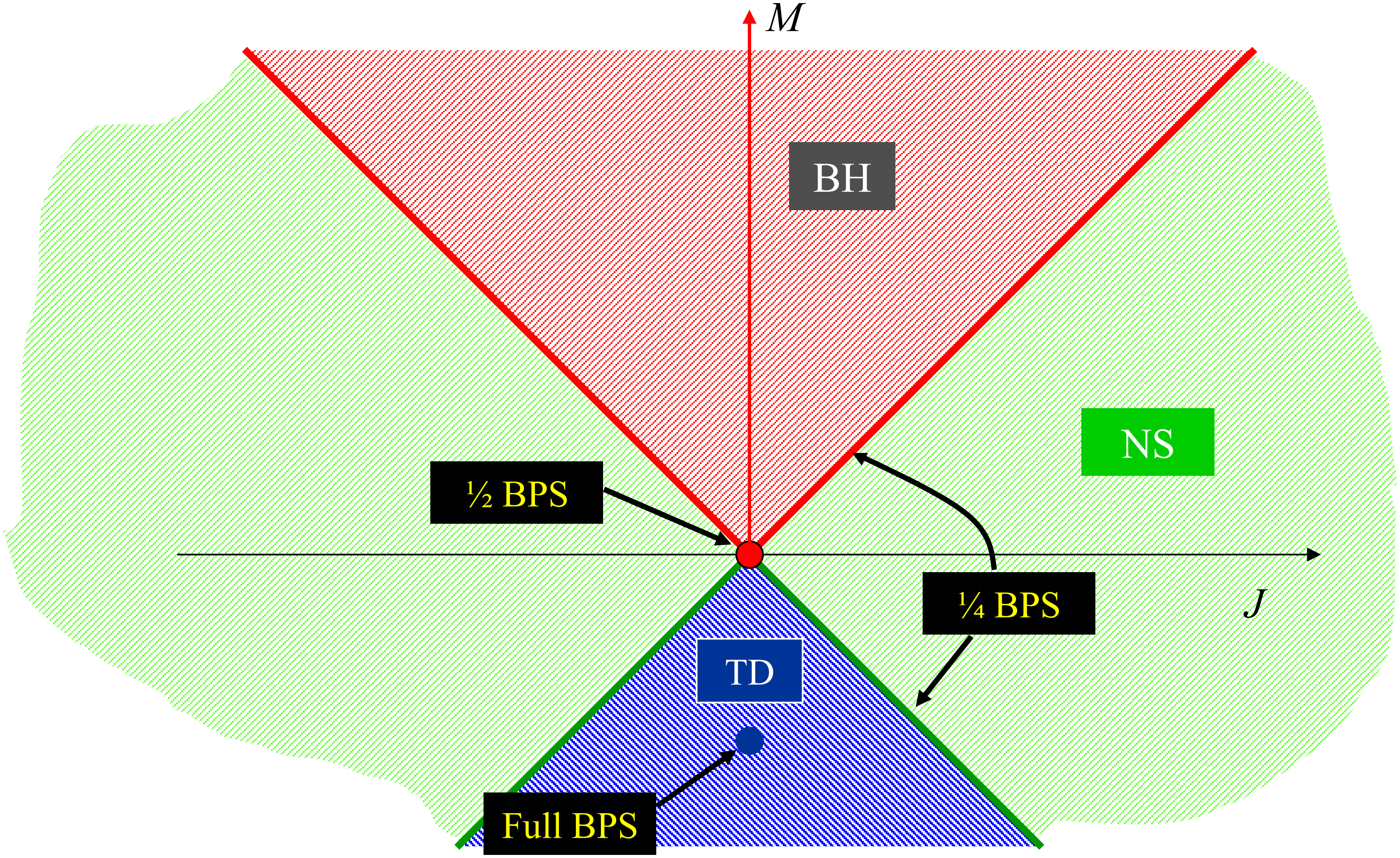
Again, this *looks like* a *2+1* black hole,

$$r_\pm^2 = \frac{M}{2}\left(1 \pm \sqrt{1 - \frac{J^2}{M^2}}\right), \quad M < 0 \to r_\pm \text{ imaginary}$$

Although this is a naked singularity, it is easy to see that for $\pm J = M \quad (< 0)$ the solution admits globally defined Killing spinors: it's a BPS state.

$$\begin{aligned} M &= -1 && \to \text{full susy} \\ M &= 0 && \to 1/2 \text{ susy} \\ \pm J = M &< 0 && \to 1/4 \text{ susy} \end{aligned}$$

For $|J| > |M|$ the NS are not easily interpreted as produced by angular defects.



M
BH
NS
½ BPS
¼ BPS
J
TD
Full BPS

4. Extensions to higher D and other theories

0-branes in $D=3$ can be easily generalized to higher dimensions.

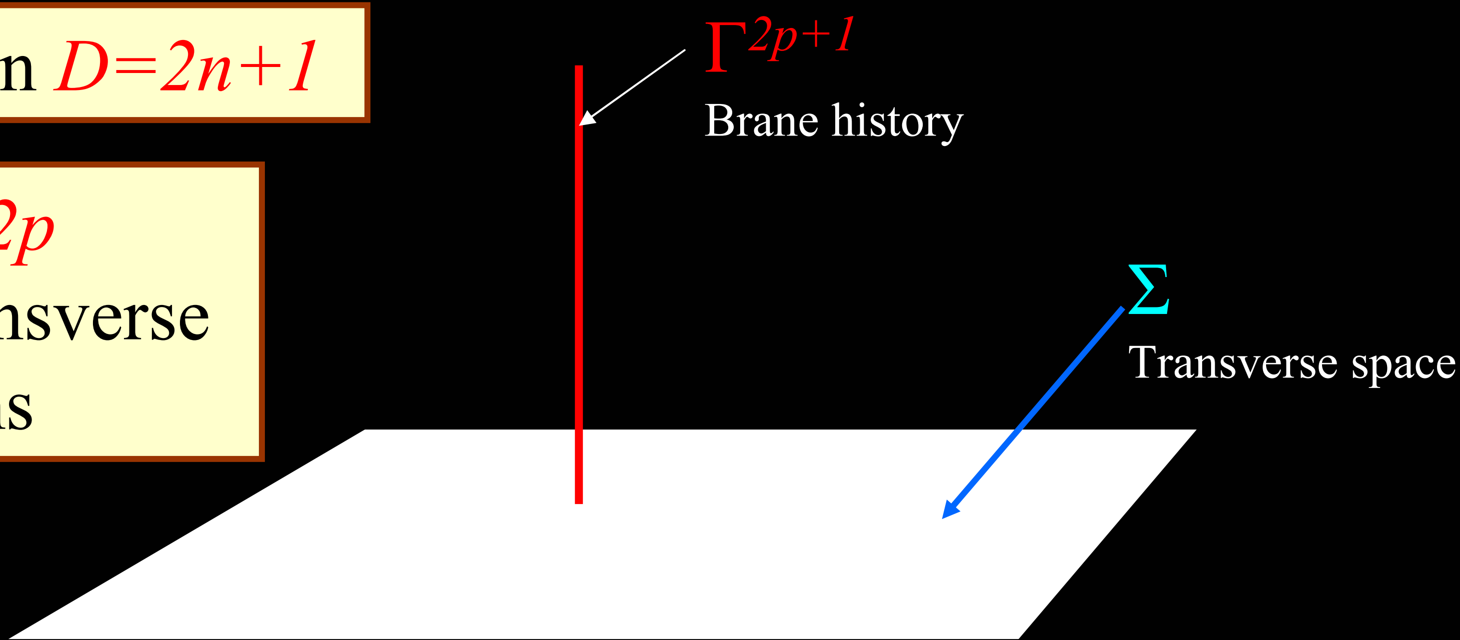
D=3	D=2n+1	Worldvolume
0-brane →	0-brane	Γ^{1}-line
0-brane →	(2n-2)-brane	Γ^{2n-1}-surface

In general, one can define the current produced by a $2p$-brane in $D=2n+1$ dimensions with

$$0 \leq p \leq n-1$$

$2p$-brane in $D=2n+1$

Σ: $m=2n-2p$ spatial transverse dimensions



Current source

$$j = 2\pi\alpha\delta(\Sigma^m)d\Omega^m_\Sigma[\varepsilon^{k_1k_2\cdots k_{m-1}k_m}J_{k_1k_2}\cdots J_{k_{m-1}k_m}]$$

Source with support at the center of Σ

AdS generators

This is a $(2n-2p)$-form that couples to a $D-2n+2p=2p+1$-form

The right coupling is:

$$\int \left\langle j \wedge C_{2p+1}(A) \right\rangle, \quad A = e^a J_a + \frac{1}{2}\omega^{ab} J_{ab}$$

where C_{2p+1} is the Chern-Simons ($2p+1$)form: Gauge invariant provided the source is covariantly conserved.

- Same recipe can be applied the to other gauge theories
- Generalizes the minimal coupling for higher dimensional branes and non abelian connections
- Provides a natural way to consistently couple CS theories to external sources

5. Summary

- 0-brane NS are a natural extension of the BH spectrum.

- They correspond to the addition of sources to gravity and other gauge theories.

- The natural and consistent coupling between branes and nonabelian connections is through the CS coupling:

$$\int \langle j \wedge C_{2p+1}(A) \rangle,$$

2(n-p)-form

2p+1-form

Nonabelian connection

Thanks!