# Holographic QCD in an Inflationary Braneworld Scenario

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# AdS/CFT

There are two major reasons to study AdS/CFT, or better gauge/gravity duality

- a definition of a string theory,
- as a tool for study nonperturbative quantum field theories via perturbative analisys in the dual gravity side.

The main example of the second reason is AdS/QCD.

Recently gauge/gravity duality has been used also in condensed matter systems: Quantum Hall, Holographic Superconductors.

# Motivations

In this work, we are interested in the use of gauge/gravity to AdS/QCD in an inflationary braneworld scenarios.

- To understand some aspects of AdS/QCD in a time-dependent backgrounds.
- Field theory in curved spacetime. Rather complicated computations using the usual methods (Textbooks by R. Wald; Birrell and Davies).
- How to describe particle creation via holography?
- Meson Spectroscopy? Meson Decay?
- How other already known features of QCD behaves in this inflationay Braneworld, such as the Cornell potential? Barosi, L., Brito, F, A. Q., JHEP, 2009, 04, 030

Ghoroku, K. et al., Phys.Rev.D, Phys.Rev.D74:124020,2006

# Braneworld Scenarios

Consider $5d$ gravity with delta function source Binetruy, P. et al., Phys. Lett., 2000, B477, 285-291; Bazeia, D. et al., Phys. Lett., 2008, B661, 179-185.

$$S = -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}(R+\Lambda_{bulk}) + \sigma\int d^5x\sqrt{-g}\,\delta(r), \qquad (1)$$

with

$$\kappa_4^2 = 8\pi G_N \simeq \kappa_5^4\sigma/6, \qquad (2)$$

where the Randall-Sundrum fine-tuning condition Randall, L. and Sundrum, R., Phys. Rev. Lett., 1999, 83, 4690-4693

$$\rho_{bulk} \equiv \Lambda_{bulk} = -\sigma^2\kappa_5^2/6 \qquad (3)$$

is satisfied.

# Inflating 3-Brane Spacetime

A general spacetime metric for a Brane Cosmology with an inflating 3-brane is given by

$$ds^2 = -n^2(t,r)dt^2 + a^2(t,r)\gamma_{ij}dx^i dx^j + b^2(t,r)dr^2, \qquad (4)$$

- $\gamma^{ij}$ is a maximally symmetric $3d$ metric with spatial curvature $k = -1, 0, 1$

3-brane inflating along $t - x^i$ directions and static along $r$ direction

- $a(t,r) = a_0(t)U(r)$
- $n(t,r) = \frac{\dot{a}(t,r)}{\dot{a}_0(t)} = U(r)$
- $b(t,r) = 1$

# Inflating 3-Brane Spacetime

The "warp factor" $U(r)$ is given by

$$U(r) = \left(-\gamma + (1+\gamma)\cosh(\mu r) - \sqrt{1+2\gamma}\,\sinh(\mu|r|)\right)^{1/2} \quad (5)$$

with

$$\gamma = \frac{3H^2}{2\sigma} \quad (6)$$

and

$$\mu = \frac{1}{3}\,\kappa_5^2\sigma \quad (7)$$

# Inflating 3-Brane Spacetime

The dynamics of the 3-Brane is given by the induced Friedmann equation Bazeia, D. et al., Phys. Lett., 2008, B661, 179-185.

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 \equiv H^2 = \frac{2}{3}\rho\left(1+\frac{\rho}{2\sigma}\right) \tag{8}$$

where $\rho_b = \rho + \sigma$, with

- $\rho$: energy density on the brane
- $\sigma$: brane tension

The metric on the 3-brane is given by

$$ds_4^2 = -dt^2 + a_0(t)^2\left(dx_1^2 + dx_2^2 + dx_3^2\right) \tag{9}$$

# Inflating 3-Brane Spacetime

The $5d$ spacetime metric is given by

$$ds_5^2 = \alpha' \left[ \frac{U^2}{R^2} \left( -dt^2 + a_0(t)^2 \left( dx_1^2 + dx_2^2 + dx_3^2 \right) \right) + \frac{R^2}{U^2 - C} \, dU^2 \right]$$

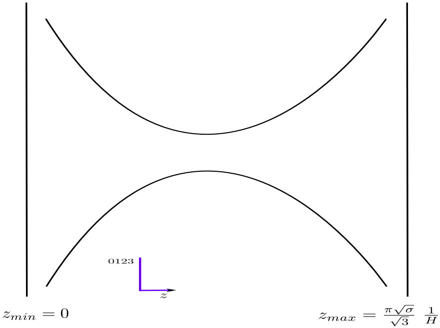
with

$$C = 2\gamma \frac{R^2}{\alpha'} \qquad\qquad R = \frac{2}{\mu}$$

where

$$a_0(t) = e^{H \ t}$$

# Inflating 3-Brane Spacetime




Setting $U = \sqrt{C}\csc(\frac{\sqrt{C}}{R}\ z)$, the metric becomes

$$ds^2 = \alpha' \frac{C}{R^2 \sin^2\left(\frac{\sqrt{C}}{R}\ z\right)} \left[-dt^2 + a_0(t)^2 \left(dx_1^2 + dx_2^2 + dx_3^2\right) + dz^2\right]$$

# Interquark Potential

Let's set Maldacena, J. M., Phys. Rev. Lett., 1998, 80, 4859-4862; Rey, S.-J. and Yee, J.-T., Eur. Phys. J., 2001, C22, 379-394; Sonnenschein, J., hep-th/0009146; Boschi-Filho, H. and Braga, N. R. F., JHEP, 2005, 03, 051

$$\begin{aligned} f(\tau,r) &= \sqrt{g_{\tau\tau}\; g_{xx}} = U(r)^2 a_0(\tau) \equiv f(r) a_0(\tau), \\ g(\tau,r) &= \sqrt{g_{\tau\tau}\; g_{rr}} = U(r) \equiv g(r), \end{aligned}$$

so that the interquark distance is

$$L(\tau) = 2\int_{r_0}^{r_1} dr\, \frac{g(\tau,r)}{f(\tau,r)} \frac{f(\tau,r_0)}{\sqrt{f(\tau,r)^2 - f(\tau,r_0)^2}} = L_0\, \frac{1}{a_0(\tau)};$$

with

$$L_0 = \int_{r_0}^{r_1} dr\, \frac{1}{U(r)} \frac{U(r_0)^2}{\sqrt{U(r)^4 - U(r_0)^4}}.$$

# Interquark Potential

The action can be written as

$$S = \frac{1}{2\pi\alpha'} \int_{r_0}^{r_1} \int_0^T dr d\tau \frac{g(r,\tau) f(r,\tau)}{\sqrt{f(r,\tau)^2 - f(r_0,\tau)^2}},$$

so that

$$S = \frac{T}{2\pi\alpha'} \int_{r_0}^{r_1} dr \frac{U(r)^3}{\sqrt{U(r)^4 - U(r_0)^4}},$$

Observe that the action is independent of $a_0(t)$.

# Interquark Potential

In summary, the integrals we have to solve are

$$L_0 = 2\frac{R^2}{U_0}\int_1^{U_1/U_0} dy\, \frac{1}{y\sqrt{y^4-1}}\,\frac{1}{\sqrt{y^2-C}}, \tag{10}$$

$$S = \frac{T\,U_0}{2\pi}\int_1^{U_1/U_0} dy\, \frac{y^3}{\sqrt{y^4-1}}\,\frac{1}{\sqrt{y^2-C}}. \tag{11}$$

in the limit $U_1 \to \infty$, where $y = U(r)/U_0$, with

$$C = 2\gamma\frac{R^2}{\alpha'} \tag{12}$$

Barosi, L., Brito, F., A.Q., JHEP, 2009, 04, 030

# The Cornell Potential

The Cornell Potential is obtained by expanding the integrals for action and length in powers of $\gamma = \frac{3H^2}{2\sigma}$. It reads Andreev, O. and Zakharov, V. I., Phys. Rev., 2006, D74, 025023

$$E(L_0) = \frac{I_1 I_2}{2\pi}\ \frac{R^2}{L_0} + \frac{I_3}{2I_1}\ \frac{\gamma}{2\pi\alpha'} L_0 + \frac{3}{16}\,\frac{1}{I_1^2}\ \frac{\gamma^2}{2\pi\alpha'^2}\ \frac{L_0^3}{R^2},$$

where

$$I_1 = \int_1^{U_1/U_0} dy \frac{1}{y^2\sqrt{y^4-1}} = 1/2\,\frac{\Gamma\left(3/4\right)^2\sqrt{2}}{\sqrt{\pi}} \overset{U_1\to\infty}{=} 0.599070117,$$

$$I_2 = \int_1^{U_1/U_0} dy \left(\frac{y^2}{\sqrt{y^4-1}} - 1\right) - 1 \overset{U_1\to\infty}{=} -I_1,$$

$$I_3 = \int_1^{U_1/U_0} dy \frac{1}{\sqrt{y^4-1}} = 1/4\,\frac{\pi^{3/2}\sqrt{2}}{\Gamma\left(3/4\right)^2} \overset{U_1\to\infty}{=} 1.311028777.$$

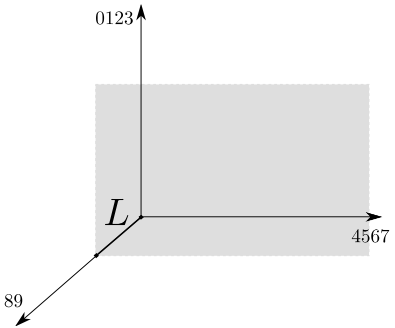
# D3-Branes

In the usual $AdS/CFT$ setup, there are $N$ D3-branes that give rise to an $\mathcal{N} = 4$ vector multiplet with

- $SU(N)$ vector bosons,
- four Weyl Fermions in the $\mathbf{4}$ of $SO(6)$,
- six scalars in the $\mathbf{6}$ of $SO(6)$.

All fields transform in the *adjoint* of $SU(N)$.
In the dual theory, this corresponds to a type $IIB$ string theory in $AdS_5 \times S^5$, where each of these factors has radius $R^2 = \sqrt{4\pi g_s N}\alpha'$.

# $N_c$ D3-Branes and $N_f$ D7-Branes




$$D3: \quad 1,2,3,------$$

$$D7: \quad 1,2,3,4,\ 5,6,7,--$$

Quark mass:

$$m_q = \frac{L}{2\pi\alpha'}$$

# $N_c$ D3-Branes and $N_f$ D7-Branes

Karch, A. and Katz, E., JHEP, 2002, 0206, 043; Erdmenger, J. et al., Eur. Phys. J., 2008, A35, 81-133.

- In the stack of $N_c$ D3-branes system is introduced a stack of $N_f$ D7-branes in the probe approximation $(N_f \ll N_c)$.
- SUSY is then broken from $\mathcal{N} = 4$ to $\mathcal{N} = 2$.
- Field content: two Weyl fermions (of opposite chirality) and two complex scalars in the fundamental of $SU(N)$
- When $L = 0$, $SO(6) \to SO(4) \times SO(2) \times SU(2)_R \times SU(2)_L \times U(1)_R$. The hypermultiplet is massless, and the R-symmetry is $SU(2)_R \times U(1)_R$.
- When $L \neq 0$, then the R-symmetry is $SU(2)_R$.

# Large Spin Mesons

We consider mesons with large $4d$ spin $J$. Kruczenski, M. et al., JHEP, 2003, 07, 049.

- Mesons are realized as spinning open strings attached to the D7-branes.
- Since the spin $J$ is large, we may obtain the meson spectrum by solving the classical equations of motion for the spinning strings. Gubser, S. et al., Nucl.Phys. B, 2002, 636, 99-114
- We work with the Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{\left(\dot{X}\cdot X'\right)^2 - \dot{X}^2 X'^2},$$

in the deformed $AdS_5$.

# Large Spin Mesons

We consider the metric to be spinning in the $x_1 - x_2$ plane. So the relevant part of the metric in suitable coordinates for this problem is

$$ds^2 = \frac{C}{R^2 \sin^2\left(\frac{\sqrt{C}}{R}\ z\right)} \left[-dx_0^2 + a_0(t)^2 \left(dx_1^2 + dx_2^2 + dx_3^2\right) + dz^2\right].$$

We consider the following ansatz:

$$\begin{aligned} x_0 &= t \\ x_1 &= \frac{\rho(\sigma)}{a_0(t)}\ \cos(\omega t) \\ x_2 &= \frac{\rho(\sigma)}{a_0(t)}\ \sin(\omega t) \\ x_3 &= \text{const.} \\ z &= z(\sigma) \end{aligned}$$

Using tha above ansatz and the rescaled coordinates

$$\tilde{\rho} = \omega\rho, \qquad \tilde{z} = \omega z$$

the action may be written as ($\xi = \frac{H}{\omega}$)

$$\begin{aligned} S &= C\xi^2\omega \int d\tau d\sigma \, \csc^2\left(\frac{\sqrt{C}\xi}{R}\tilde{z}\right) \cdot \\ &\cdot \sqrt{\tilde{z}'^2 \left(1 - \left(\xi^2 + 1\right)\tilde{\rho}^2\right) + \left(1 - \tilde{\rho}^2\right)\tilde{\rho}'^2} \end{aligned}$$

and the energy and spin are

$$\begin{aligned} E &= C^2\xi^4\omega^4 \int d\sigma \, \frac{\left(\tilde{z}'^2 + \tilde{\rho}'^2\right)\csc^4\left(\frac{\sqrt{C}\xi}{R}\tilde{z}\right)}{\mathcal{L}} \\ J &= C^2\xi^4\omega^2 \int d\sigma \, \frac{\tilde{\rho}^2\left(\tilde{z}'^2 + \tilde{\rho}'^2\right)\csc^4\left(\frac{\sqrt{C}\xi\tilde{z}}{R}\right)}{\mathcal{L}} \end{aligned}$$

We may fix one of two convenient gauge: either $\tilde{\rho} = \sigma$ or $\tilde{z} = \sigma$.
In the gauge $\tilde{\rho} = \sigma$, the equations of motion becomes

$$\frac{\tilde{z}''}{\frac{\tilde{\rho}^2-1}{(1+\xi^2)\tilde{\rho}^2-1} + \tilde{z}'^2} + \frac{2}{\tilde{z}} - \\ -\frac{\tilde{\rho}\tilde{z}'}{1-\tilde{\rho}}\left[1+\xi^2\left(1-\frac{1}{((1+\xi^2)\tilde{\rho}^2-1)\left(\frac{\tilde{\rho}^2-1}{(1+\xi^2)\tilde{\rho}^2-1}+\tilde{z}'^2\right)}\right)\right] = 0$$

In the limit $\xi \equiv \frac{H}{\omega} \to 0$, this equation becomes

$$\frac{\tilde{z}''}{1+\tilde{z}'^2} + \frac{2}{\tilde{z}} - \frac{\tilde{\rho}\ \tilde{z}'}{1-\tilde{\rho}^2} = 0$$

Kruczenski, M. et al., JHEP, 2003, 07, 049.

# Results: $T_{eff} = \frac{1}{2\pi\alpha'_{eff}}$

For $\omega > H/z_{D7}$, we obtain

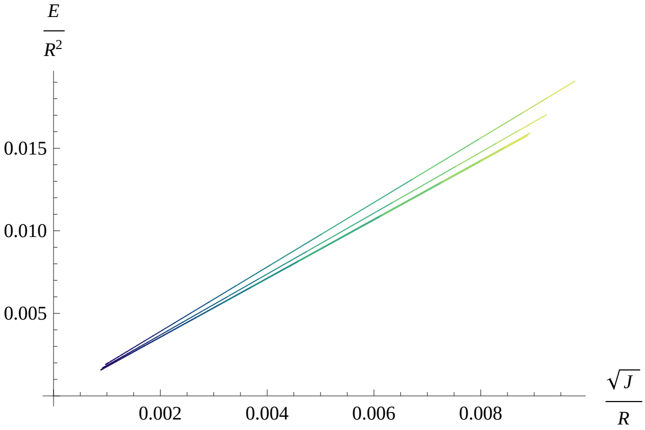
$$E = \frac{1}{\xi^2 z_{D7}} \int d\sigma \frac{\pi R^2 + \frac{1}{3} C z_{D7}^2}{\sqrt{1-\sigma^2}} \quad (13)$$

$$J = \frac{w}{\xi^2 z_{D7}} \int d\sigma \sigma^2 \ \frac{\pi R^2 + \frac{1}{3} C z_{D7}^2}{\sqrt{1-\sigma^2}} \quad (14)$$

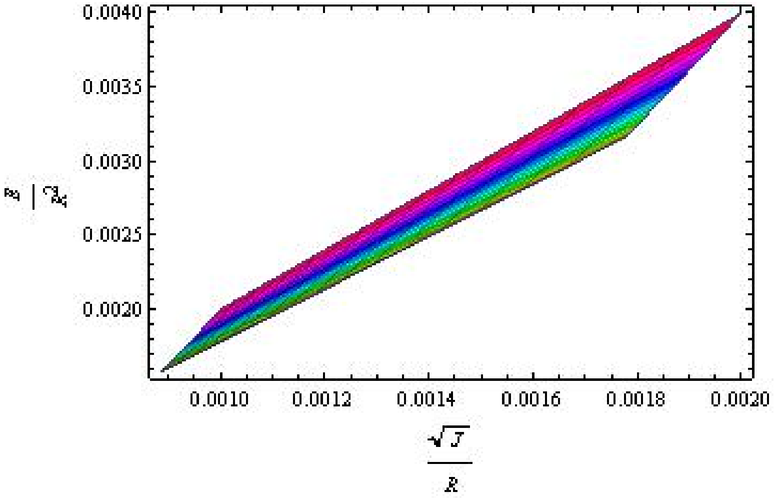
We obtain a Regge behaviour $E = \alpha'_{eff}\sqrt{J}$, with

$$\alpha'_{eff} = \frac{\pi R^2 + \frac{1}{3} C z_{D7}^2}{z_{D7}} \quad (15)$$

# Results




Curves for distincts values of $H$.


0.0040
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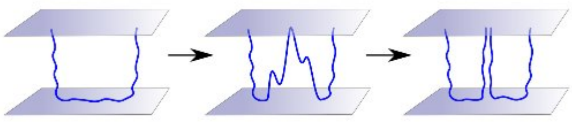
0.0020

# Meson Decay



Peeters, K. and Zamaklar, M., *The string/gauge theory correspondence in QCD*, Eur.Phys.J.ST, 2007, 152:113-138,2007

- The string splitting probability is given by

$$\mathcal{P} = \mathcal{P}_{\text{fluct.}} \times \mathcal{P}_{\text{split}}$$

- $\mathcal{P}_{\text{split}} \equiv \mathcal{P}_{\text{split}}(T_{\text{eff}}, m_q)$
- $\mathcal{P}_{\text{fluct.}} \equiv \mathcal{P}_{\text{fluct.}}(\Psi\left[\{\eta_n\}\right])$

# Thanks a lot!