New trends in Quatum Gravity II,
Sao Paulo, September 2009

STRING COMPACTIFICATIONS, FLUXES AND SUPERGRAVITY

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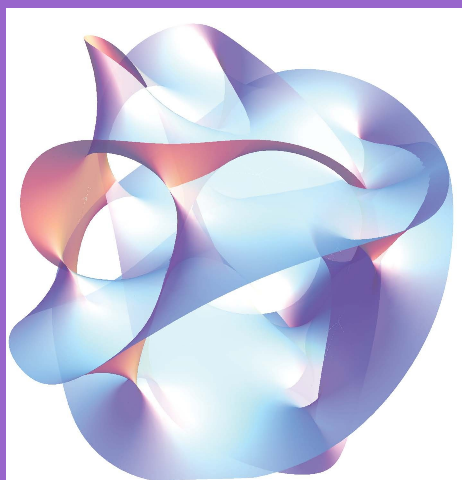
Based on G.A, Camara, Font, Ibanez (JHEP 0605:070,2006.)
G.A , Camara, Rosabal (Nucl.Phys.B814:21-52,2009.)
G.A, M. Graña, M. Petrini (in progress)

LINK BETWEEN STRING THEORY AND D=4 WORLD?



?

D=4 (broken) **SUGRA** including the **STANDARD MODEL**

STRING PHENOMENOLOGY

- Link between String Theory and Particle Physics Or: How is the Standard Model embedded in String Theory ?

- The String Model-Building Program aims at a general study of possible compactifications/constructions giving rise to a low-energy theory resembling as much as possible the SM (or the MSSM) in the framework of String theory.

- On the journey we may identify general patterns (e.g. symmetries, extra particle content etc.) that might be present in large classes of realistic vacua.
Discard certain non-stringy model building approaches.

- Find inspiring ideas to solve old problems like SCP violation, unification, fermion masses an mixings, dark matter/energy,...

STRING MODEL BUILDING

- < 1995 Top-Bottom Scenario

$D = 10$ **HETEROTIC STRING**

$D = 4$

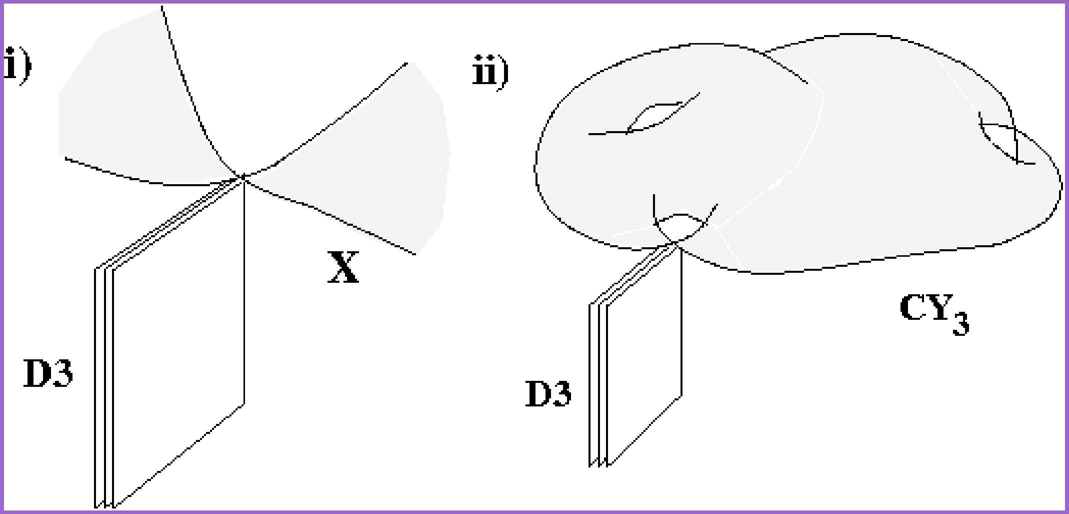
Intensive work: CY, Orbifolds, Gepner's Models, Free Fermions

- $\geq$ 1995 Bottom-up Scenario

Gauge interactions localized on extended Dp-branes

G. A, L. E. Ibañnez, F. Quevedo, A. M. Uranga, ,2000
D. Berenstein, V. Jejjala and R.G. Leigh, 2001
....

BRANES AT SINGULARITIES



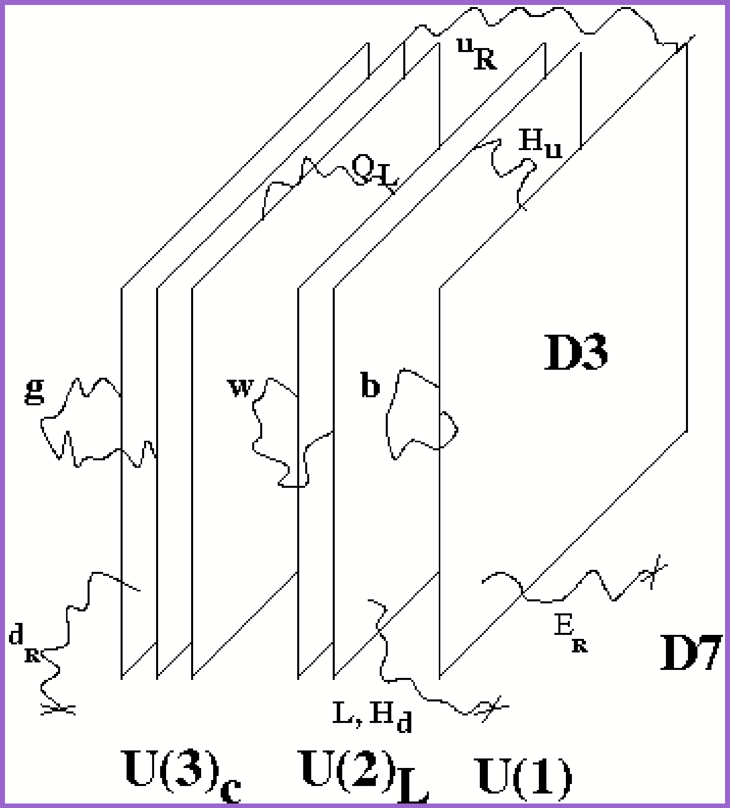
At singularity

$$C^3/\mathbb{Z}_M \qquad g_M = (\theta_v, \gamma) \qquad |\Psi > \Lambda = |\theta\Psi > \gamma^{-1}\Lambda\gamma$$

$$|\Psi^{\mu}_{-\frac{1}{2}} > \Lambda^0 = |\Psi^{\mu}_{-\frac{1}{2}}\rangle\gamma^{-1}\Lambda^0\gamma$$

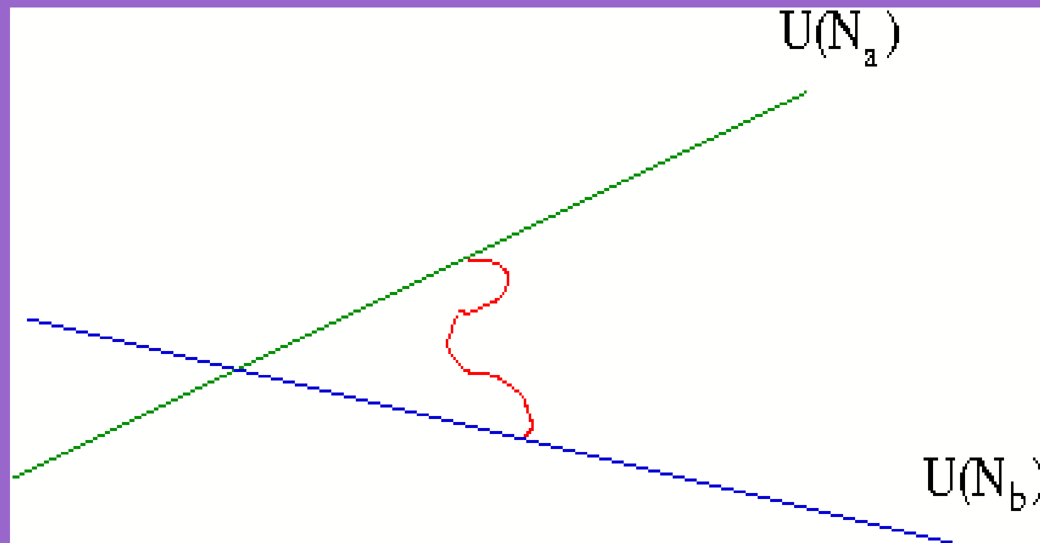
$$|s_{st}, s_1, s_2, s_3\rangle\Lambda^0 = \mathrm{e}^{2i\pi v\cdot s}|s_{st}, s_1, s_2, s_3,\rangle\gamma\Lambda^0\gamma^{-1}$$

STANDARD LIKE MODEL



R. Blumenhagen, L. G¨orlich, B. K¨ors and D. L¨ ust, (2000)
C. Angelantonj, I. Antoniadis, E. Dudas, A. Sagnotti, (2000)
G. A, S. Franco, L. E. Ibanez, R. Rabadan, A. M. Uranga,(2000)

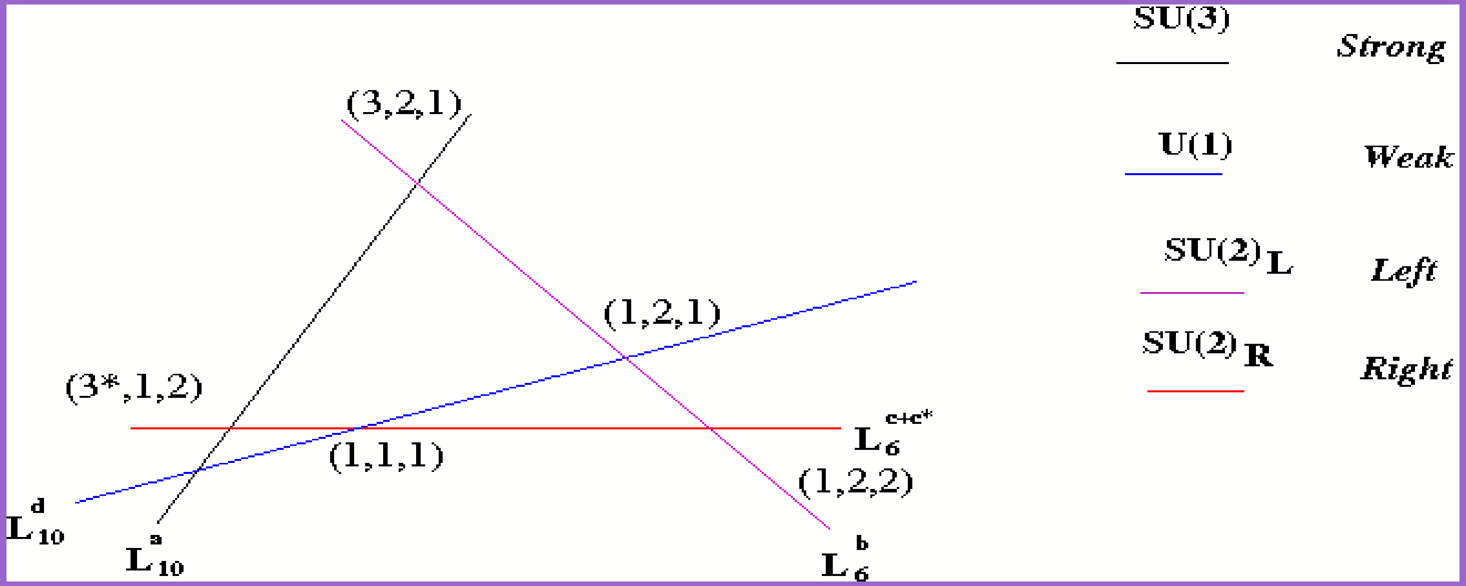
INTERSECTING BRANES

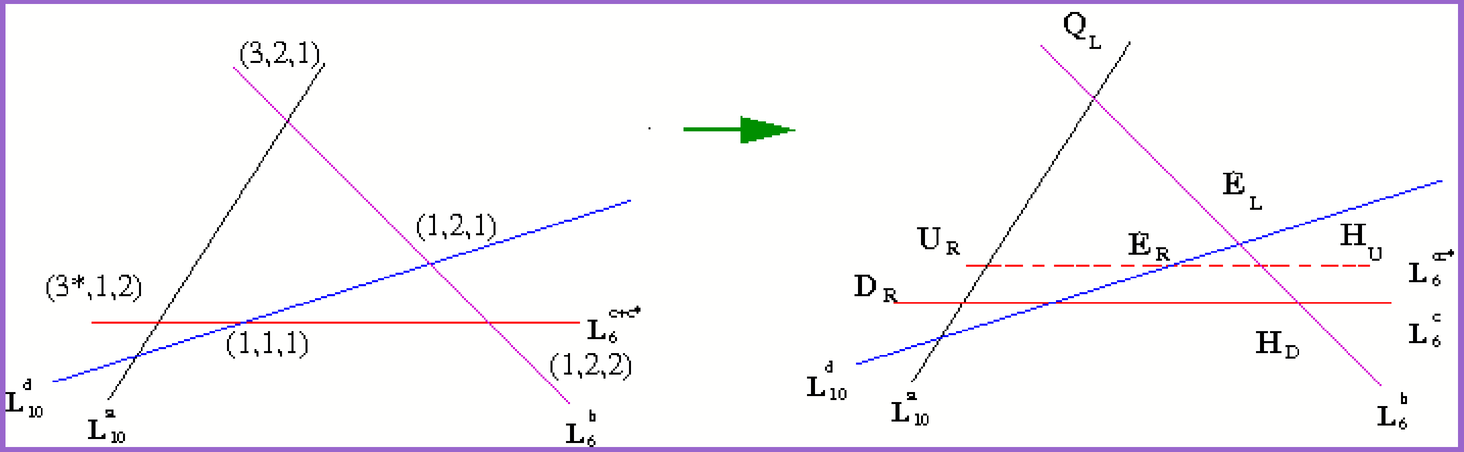


$$\prod_{a=1}^{K} U(N_a)$$

$$\sum_{a<b} I_{ab}\,(N_a, \overline{N}_b)$$

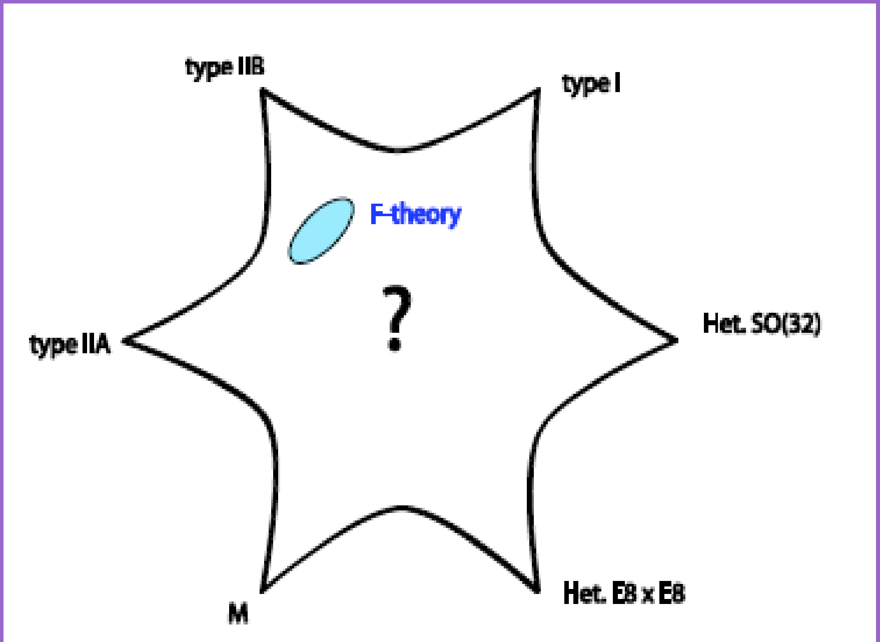
$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_i (n_a^i m_b^i - m_a^i n_b^i)$$





STANDARD LIKE MODEL

F-Theory compactifications:



- Local GUT models [Vafa et al.; Donagi et al.; ... '08
- Exceptional groups and spinorial representations
- Etc.
- **Interpretation of many of the fluxes**

PROBLEMS:

In spite of recent progress:

- Fully realistic model must still be found

- How do we select among all (LANDSCAPE) possible models?

FLUX COMPACTIFICATIONS

may help.

→

MODULI STABILIZATION
SUSY BREAKING
NEW POSSIBILITIES

MODULI

Type IIB

$$\left\{ \begin{array}{ll} \mathrm{NS-NS} & G_{MN}, B_2, \Phi \\ \mathrm{R-R} & C_0, C_2, C_4 \end{array} \right\}$$

Field strenght:

$$F^{(p)} = dC^{(p-1)}$$

Type IIA

$$\left\{ \begin{array}{ll} \mathrm{NS-NS} & G_{MN}, B_2, \Phi \\ \mathrm{R-R} & C_1, C_3 \end{array} \right\}$$

Compactification: $M \rightarrow \{\mu = 0, 1, 2, 3\} + \{m = 4, \ldots 9\}$

Scalars Φ, G_{mn}, C_{mn}

Moduli: VEV's are not fixed.
coupling(dilaton), Khaler structure(size of 2-cycles), Complex structure(size of 3-cycles)

FLUXES

$$\frac{1}{(2\pi\sqrt{\alpha'})^{p-1}}\int_{\Sigma_p}\hat{F}_p = m_p \in \mathbb{Z}$$

$$d * \hat{F} = 0$$ E.M (no source)

$$d\hat{F} = 0$$ Bianchi identity

+ supersymmetry conditions (i.e $*_6\mathbf{F}^{(3)} = i\mathbf{F}^{(3)}$)

For fixed set of fluxes m_p geometrical moduli cannot be changed arbitrarily

MODULI are stabilized

→ **Superpotential**

Gukov,Vafa,Witten (2000)
....

Also, why exciting only the metric? Other allowed configurations must be explored

Type IIB orientifold (O9): $\otimes_{j=1}^{3} T_j^2/\Omega_P(-1)^{F_L}$

Supersymmetric compactif cation: Ω, J

In terms of complexif ed forms

$$
\begin{aligned}
\Omega &= (dx^1 + iU_1\, dx^4) \wedge (dx^2 + iU_2\, dx^5) \wedge (dx^3 + iU_3\, dx^6) \\
&= \alpha_0 + \frac{1}{2} \sum_{r \neq s \neq t} \alpha_r U_s U_t - i\beta_0 U_1 U_2 U_3 + i \sum_r U_r \beta_r \\
J_c &= -iT_i w_i \\
\omega_i &= -dx^i \wedge dy^i
\end{aligned}
$$

CLOSED MODULI

	dim.	form	dual form	moduli
Kähler	$h^-_{(1,1)}$	ω_A	$\widetilde{\omega}_A$	$T_A = -i\int J_c \wedge \widetilde{\omega}_A$
c.s + dilaton	$1 + h_{(1,2)}$	α_L	β_L	$U_L = i\int \Omega_c \wedge \beta_L$

Type IIB fluxes (O9)

Turn on RR and NSNS f uxes

$$\overline{F}_3 = -m\beta_0 - e_0\alpha_0 + \sum_{i=1}^{3}(e_i\beta_i - q_i\alpha_i) \ .$$

Geometric f uxes (Scherk-Shcwarz)

$$ds^2 = \sum_k (dx^k + \omega^k_{pq} x^q dx^p)^2 \qquad (\omega^k_{pq} \equiv h_i, \bar{h}_i, b_{ij}, \bar{b}_{ij})$$

Correspond to $$d\eta^p = -\tfrac{1}{2}\omega^p_{mn}\eta^m \wedge \eta^n .$$

Bianchi identity :

$$d^2 = 0 \quad \Rightarrow \quad \omega^p_{[ab}\omega^l_{c]p} = 0$$

constraints

Type IIB O9 superpotential:

Gukov,Vafa,Witten (2000)

$$
\begin{aligned}
W_{O9} &= \int_{\mathrm{T}^6} \Omega \wedge (\overline{\mathbf{F}}_3 + \omega J_c) \\
&= \left[-ie_0 - \sum_{i=1}^{3} h_i T_i\right] U_1 U_2 U_3 \\
&+ \frac{1}{2} \sum_{r\neq s\neq t} \left[\Big(+ e_r + i \sum_{j=1}^{3} b_{rj} T_j\right] U_s U_t \\
&+ \sum_{i=1}^{3} i q_i + \sum_{j=1}^{3} \bar{b}_{ij} T_j \Bigg] U_i + \left[m - i \sum_{j=1}^{3} \bar{h}_j T_j\right]
\end{aligned}
$$

↓

Scalar potential

Some Moduli will be stabilized

O9

$$W_{O9} = \int_{\mathrm{T}^6} \Omega \wedge (\overline{\mathbf{F}}_3 + \omega J_c)$$

$$= \left(-ie_0 - \sum_{i=1}^{3} h_i T_i\right) U_1 U_2 U_3 + \frac{1}{2} \sum_{r \neq s \neq t} \left(e_r + i \sum_{j=1}^{3} b_{rj} T_j\right) U_s U_t$$

$$+ \sum_{i=1}^{3} \left(iq_i + \sum_{j=1}^{3} \bar{b}_{ij} T_j\right) U_i + m - i \sum_{j=1}^{3} \bar{h}_j T_j$$

O3

$$W_{O3} = \int_{\mathrm{T}^6} (\overline{\mathcal{F}}_3 - iS\overline{\mathcal{H}}_3) \wedge \Omega$$

$$= e_0 + i \sum_{i=1}^{3} e_i U_i - q_1 U_2 U_3 - q_2 U_1 U_3 - q_3 U_1 U_2 + imU_1 U_2 U_3$$

$$+ S\left[ih_0 - \sum_{i=1}^{3} a_i U_i + i\bar{a}_1 U_2 U_3 + i\bar{a}_2 U_1 U_3 + i\bar{a}_3 U_1 U_2 - \bar{h}_0 U_1 U_2 U_3\right] .$$

Fluxes missing on both sides

O9 Is not T-dual of **O3**

$$U_i \leftrightarrow \frac{1}{U_i}; T_i \leftrightarrow T_i; S \leftrightarrow S$$

$$F_{MN_1 \cdots N_p} \overset{\mathrm{T_M}}{\longleftrightarrow} F_{N_1 \cdots N_p}$$

Buscher rule

T-duality invariance requires the introduction of new fluxes

[Shelton, Taylor Wetch
G.A, P.Cámara, A. Font, L. Ibañez]

$$-\overline{H}_{MNP} \overset{T_M}{\longleftrightarrow} \omega^{M}_{NP} \overset{T_N}{\longleftrightarrow} Q^{MN}_{P} \overset{T_P}{\longleftrightarrow} -R^{MNP}$$

IIB/O3	IIA/O6	IIB/O9	flux
$\overline{\mathcal{H}}_{123}$	R^{123}	$\mathbf{R}^{123}$	$\bar{h}_0$
$\overline{\mathcal{H}}_{423}$	$-Q_4^{23}$	$\mathbf{R}^{423}$	$-\bar{a}_1$
$\overline{\mathcal{H}}_{153}$	$-Q_5^{31}$	$\mathbf{R}^{153}$	$-\bar{a}_2$
$\overline{\mathcal{H}}_{126}$	$-Q_6^{12}$	$\mathbf{R}^{126}$	$-\bar{a}_3$
$\overline{\mathcal{H}}_{156}$	$-\omega_{56}^{1}$	$\mathbf{R}^{156}$	$-a_1$
$\overline{\mathcal{H}}_{426}$	$-\omega_{64}^{2}$	$\mathbf{R}^{426}$	$-a_2$
$\overline{\mathcal{H}}_{453}$	$-\omega_{45}^{3}$	$\mathbf{R}^{453}$	$-a_3$
$\overline{\mathcal{H}}_{456}$	$\overline{H}_{456}$	$\mathbf{R}^{456}$	h_0

Table 1: NS IIB/O3 fluxes and their T-duals.

IIB/O3	IIA/O6	IIB/O9	flux	IIB/O3	IIA/O6	IIB/O9	flux
$F_{123} = -\tilde{F}^{456}$	F_0	$-\boldsymbol{F}_{456}$	$-m$	$H_{123} = -\tilde{H}^{456}$	R^{123}	$\boldsymbol{R}^{123}$	$\bar{h}_0$
$F_{423} = \tilde{F}^{156}$	F_{14}	$\boldsymbol{F}_{156}$	$-q_1$	$H_{423} = \tilde{H}^{156}$	$-Q_4^{23}$	$\boldsymbol{R}^{423}$	$-\bar{a}_1$
$F_{153} = \tilde{F}^{426}$	F_{25}	$\boldsymbol{F}_{426}$	$-q_2$	$H_{153} = \tilde{H}^{426}$	$-Q_5^{31}$	$\boldsymbol{R}^{153}$	$-\bar{a}_2$
$F_{126} = \tilde{F}^{453}$	F_{36}	$\boldsymbol{F}_{453}$	$-q_3$	$H_{126} = \tilde{H}^{453}$	$-Q_6^{12}$	$\boldsymbol{R}^{126}$	$-\bar{a}_3$
$F_{156} = -\tilde{F}^{423}$	F_{2536}	$-\boldsymbol{F}_{423}$	e_1	$H_{156} = -\tilde{H}^{423}$	$-\omega^1_{56}$	$\boldsymbol{R}^{156}$	$-a_1$
$F_{426} = -\tilde{F}^{153}$	F_{1436}	$-\boldsymbol{F}_{153}$	e_2	$H_{426} = -\tilde{H}^{153}$	$-\omega^2_{64}$	$\boldsymbol{R}^{426}$	$-a_2$
$F_{453} = -\tilde{F}^{126}$	F_{1425}	$-\boldsymbol{F}_{126}$	e_3	$H_{453} = -\tilde{H}^{126}$	$-\omega^3_{45}$	$\boldsymbol{R}^{453}$	$-a_3$
$F_{456} = \tilde{F}^{123}$	F_{142536}	$\boldsymbol{F}_{123}$	$-e_0$	$H_{456} = \tilde{H}^{123}$	$\overline{H}_{456}$	$\boldsymbol{R}^{456}$	h_0

Table 1: Type IIB/O3 3-form RR (left) and NSNS (right) fluxes and their T-duals.

IIB/O3	IIA/O6	IIB/O9	flux
$\begin{pmatrix} Q_4^{23} & Q_5^{31} & Q_6^{12} \end{pmatrix}$	$-\begin{pmatrix} \overline{H}_{423} & \overline{H}_{153} & \overline{H}_{126} \end{pmatrix}$	$\begin{pmatrix} \omega^4_{23} & \omega^5_{31} & \omega^6_{12} \end{pmatrix}$	$-\begin{pmatrix} h_1 & h_2 & h_3 \end{pmatrix}$
$\begin{pmatrix} -Q_1^{23} & Q_5^{34} & Q_6^{42} \\ Q_4^{53} & -Q_2^{31} & Q_6^{15} \\ Q_4^{26} & Q_5^{61} & -Q_3^{12} \end{pmatrix}$	$\begin{pmatrix} -\omega^1_{23} & \omega^4_{53} & \omega^4_{26} \\ \omega^5_{34} & -\omega^2_{31} & \omega^5_{61} \\ \omega^6_{42} & \omega^6_{15} & -\omega^3_{12} \end{pmatrix}$	$\begin{pmatrix} -\omega^1_{23} & \omega^5_{34} & \omega^6_{42} \\ \omega^4_{53} & -\omega^2_{31} & \omega^6_{15} \\ \omega^4_{26} & \omega^5_{61} & -\omega^3_{12} \end{pmatrix}$	$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$
$\begin{pmatrix} Q_1^{56} & Q_2^{64} & Q_3^{45} \end{pmatrix}$	$-\begin{pmatrix} R^{156} & R^{426} & R^{453} \end{pmatrix}$	$\begin{pmatrix} \omega^1_{56} & \omega^2_{64} & \omega^3_{45} \end{pmatrix}$	$-\begin{pmatrix} \bar{h}_1 & \bar{h}_2 & \bar{h}_3 \end{pmatrix}$
$\begin{pmatrix} -Q_4^{56} & Q_2^{61} & Q_3^{15} \\ Q_1^{26} & -Q_5^{64} & Q_3^{42} \\ Q_1^{53} & Q_2^{34} & -Q_6^{45} \end{pmatrix}$	$\begin{pmatrix} -Q_4^{56} & Q_1^{26} & Q_1^{53} \\ Q_2^{61} & -Q_5^{64} & Q_2^{34} \\ Q_3^{15} & Q_3^{42} & -Q_6^{45} \end{pmatrix}$	$\begin{pmatrix} -\omega^4_{56} & \omega^2_{61} & \omega^3_{15} \\ \omega^1_{26} & -\omega^5_{64} & \omega^3_{42} \\ \omega^1_{53} & \omega^2_{34} & -\omega^6_{45} \end{pmatrix}$	$\begin{pmatrix} \bar{b}_{11} & \bar{b}_{12} & \bar{b}_{13} \\ \bar{b}_{21} & \bar{b}_{22} & \bar{b}_{23} \\ \bar{b}_{31} & \bar{b}_{32} & \bar{b}_{33} \end{pmatrix}$

Table 2: Type IIB/O3 NSNS Q-fluxes and their T-duals.

T-duality invariant superpotential

$$\mathcal{W} = \int_{\mathrm{T}^6} \left[(\overline{\mathcal{F}}_3 - iS\overline{\mathcal{H}}_3) + \mathcal{Q}\mathcal{J}_c \right] \wedge \Omega \ .$$

S duality

T, S-duality invariant superpotential

$$\mathcal{W} = \int_{\mathrm{T}^6} \left[(\overline{\mathcal{F}}_3 - iS\overline{\mathcal{H}}_3) + (\mathcal{Q} - iS\mathcal{P})\mathcal{J}_c \right] \wedge \Omega \ .$$

S duality

$$\begin{pmatrix} \overline{\mathcal{F}}_3 \\ \overline{\mathcal{H}}_3 \end{pmatrix} \to \begin{pmatrix} k & \ell \\ m & n \end{pmatrix} \begin{pmatrix} \overline{\mathcal{F}}_3 \\ \overline{\mathcal{H}}_3 \end{pmatrix} \cdot \qquad \begin{pmatrix} \mathcal{Q} \\ \mathcal{P} \end{pmatrix} \to \begin{pmatrix} k & \ell \\ m & n \end{pmatrix} \begin{pmatrix} \mathcal{Q} \\ \mathcal{P} \end{pmatrix} \cdot$$

$$S \to \frac{kS - i\ell}{imS + n}$$

$$SL(2, \mathbb{Z})$$

[G.A, P.Cámara, A. Font, L. Ibañez]

$SL(2,\mathbb{Z})^7$

Flux parameter	Weight	Flux parameter	Weight
$\bar{h}'_0$	$(-,-,-,-,-,-,-)$	e_0	$(+,+,+,+,+,+,+)$
h_0	$(-,+,+,+,+,+,+)$	m'	$(+,-,-,-,-,-,-)$
$-h_i$	$(+,\overbrace{-,+,+}^{i},+,+,+)$	$-\bar{f}'_i$	$(-,\overbrace{+,-,-}^{i},-,-,-)$
e_j	$(+,+,+,+,\overbrace{-,+,+}^{j})$	$\bar{a}'_j$	$(-,-,-,-,\overbrace{+,-,-}^{j})$
$\bar{h}'_i$	$(+,\overbrace{+,-,-}^{i},-,-,-)$	f_i	$(-,\overbrace{-,+,+}^{i},+,+,+)$
q'_j	$(+,-,-,-,\overbrace{+,-,-}^{j})$	a_j	$(-,+,+,+,\overbrace{-,+,+}^{j})$
$\bar{g}'_{ji}$	$(-,\overbrace{+,-,-}^{i},\overbrace{+,-,-}^{j})$	b_{ji}	$(+,\overbrace{-,+,+}^{i},\overbrace{-,+,+}^{j})$
a'_j	$(-,-,-,-,\overbrace{-,+,+}^{j})$	q_j	$(+,+,+,+,\overbrace{+,-,-}^{j})$
$-g_{ji}$	$(-,\overbrace{-,+,+}^{i},\overbrace{-,+,+}^{j})$	$-\bar{b}'_{ji}$	$(+,\overbrace{+,-,-}^{i},\overbrace{+,-,-}^{j})$
$-\bar{a}_j$	$(-,+,+,+,\overbrace{+,-,-}^{j})$	$-e'_j$	$(+,-,-,-,\overbrace{-,+,+}^{j})$
$-\bar{b}_{ji}$	$(+,\overbrace{-,+,+}^{i},\overbrace{+,-,-}^{j})$	$-g'_{ji}$	$(-,\overbrace{+,-,-}^{i},\overbrace{-,+,+}^{j})$
$-m$	$(+,+,+,+,-,-,-)$	$-h'_0$	$(-,-,-,-,+,+,+)$
b'_{ji}	$(+,\overbrace{+,-,-}^{i},\overbrace{-,+,+}^{j})$	$\bar{g}_{ji}$	$(-,\overbrace{-,+,+}^{i},\overbrace{+,-,-}^{j})$
f'_i	$(-,\overbrace{+,-,-}^{i},+,+,+)$	$\bar{h}_i$	$(+,\overbrace{-,+,+}^{i},-,-,-)$
$-e'_0$	$(+,-,-,-,+,+,+)$	$-\bar{h}_0$	$(-,+,+,+,-,-,-)$
$-\bar{f}_i$	$(-,\overbrace{-,+,+}^{i},-,-,-)$	$-h'_i$	$(+,\overbrace{+,-,-}^{i},+,+,+)$

Table 1: Spinorial embedding of the background fluxes. The weights in each column correspond to one of the two Weyl spinors on which the set of fluxes $\mathbb{G}$ can be decomposed.

Moduli	Weight
S	$(1,0,0,0,0,0,0)$
T_i	$(0,\overbrace{1,0,0}^{i},0,0,0)$
U_i	$(0,0,0,0,\overbrace{1,0,0}^{i})$

Table 1: Embedding of the moduli in a **7** of $SL(2,\mathbb{Z})^7$.

$$
\begin{aligned}
W_{Flux} \quad = \quad & e_0 - i\sum_{i=1}^{3} h_i T_i + \frac{1}{2}\sum_{l\neq m\neq n} h'_l T_m T_n + i e'_0 T_1 T_2 T_3 \\
+ \quad & \left(i h_0 - \sum_{i=1}^{3} f_i T_i - \frac{i}{2}\sum_{l\neq m\neq n} f'_l T_m T_n - h'_0 T_1 T_2 T_3 \right) S \\
+ \quad & \sum_{i=1}^{3} \left[\left(- a_i + i\sum_{j=1}^{3} g_{ij} T_j - \frac{1}{2}\sum_{l\neq m\neq n} g'_{il} T_m T_n + i a'_i T_1 T_2 T_3 \right) S \right. \\
+ \quad & \left. i e_i - \sum_{j=1}^{3} b_{ij} T_j - \frac{i}{2}\sum_{l\neq m\neq n} b'_{il} T_m T_n - e'_i T_1 T_2 T_3 \right] U_i \\
+ \quad & \frac{1}{2}\sum_{r\neq s\neq t} \left[\left(i\bar{a}_r + \sum_{j=1}^{3} \bar{g}_{rj} T_j + \frac{i}{2}\sum_{l\neq m\neq n} \bar{g}'_{rl} T_m T_n - \bar{a}'_r T_1 T_2 T_3 \right) S \right. \\
- \quad & \left. q_r + i\sum_{j=1}^{3} \bar{b}_{rj} T_j - \frac{1}{2}\sum_{l\neq m\neq n} \bar{b}'_{rl} T_m T_n + i q'_r T_1 T_2 T_3 \right] U_s U_t \\
+ \quad & \left[- \left(\bar{h}_0 + i\sum_{j=1}^{3} \bar{f}_j T_j - \frac{1}{2}\sum_{l\neq m\neq n} \bar{f}'_l T_m T_n + i\bar{h}'_0 T_1 T_2 T_3 \right) S \right. \\
+ \quad & \left. i m + \sum_{j=1}^{3} \bar{h}_j T_j + \frac{i}{2}\sum_{l\neq m\neq n} \bar{h}'_l T_m T_n - m' T_1 T_2 T_3 \right] U_1 U_2 U_3
\end{aligned}
$$

Fluxes are constrained

Bianchi identities

Tadpole cancellation

Tadpoles:

$$\int -C_8 \wedge QF_3 + \tilde{C}_8 \wedge PH_3 + C_8' \wedge (QH_3 + PF_3)$$

Source F-theory (p,q) seven branes

$$(QH_3 + PF_3)_k = 2\sum_I p_k^I q_k^I$$

$$(QF_3)_k = \sum_I (p_k^I)^2 \ ,$$

$$(PH_3)_k = \sum_I (q_k^I)^2$$

- **What is the 10D interpretation of fluxes?**
- **How do fluxes manifest in 4D ?**
- **Restrictions on fluxes: BI, Tadpoles**

T-duality

Circle

$$\alpha' M^2 = \alpha' [\left(\frac{m}{R} \right)^2 + \left(\frac{Rn}{\alpha'} \right)^2] + 2N - 4)$$

$$R \quad \rightarrow \quad \tilde{R} = \frac{\alpha'}{R}$$

$$n \quad \rightarrow \quad m$$

Torus

$$T \quad \propto \quad A = R_x R_y$$

$$U \quad \propto \quad R_x / R_y$$

$\Rightarrow$ $T \leftrightarrow U$

T-duality example:

$$S = S_g + S_B + S_\Phi,$$

$$S_g = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}g_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu,$$

$$S_B = \frac{1}{4\pi\alpha'}\int d^2\sigma\varepsilon^{\alpha\beta}B_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu,$$

$$S_\Phi = \frac{1}{4\pi}\int d^2\sigma\sqrt{-h}\Phi R^{(2)}.$$

Lagrange multiplier $\tilde{X}^9 \rightarrow \epsilon^{\alpha\beta}\partial_\alpha V_\beta = 0 \rightarrow V_\beta = \partial_\beta X^9$

$$4\pi\alpha' S = \int d^2\sigma\left[\sqrt{-h}h^{\alpha\beta}\left(-g_{99}V_\alpha V_\beta - 2g_{9\mu}V_\alpha\partial_\beta X^\mu - g_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu\right)+\right.$$

$$\left.\varepsilon^{\alpha\beta}(B_{9\mu}V_\alpha\partial_\beta X^\mu + B_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu) + \widetilde{X}^9\varepsilon^{\alpha\beta}\partial_\alpha V_\beta + \alpha'\sqrt{-h}R^{(2)}\Phi(X)\right]. \quad (6.94)$$

$\rightarrow S$

Using equations of motion for V_α

$$\tilde{S} = S_{\tilde{g}} + S_{\tilde{B}} + S_{\tilde{\Phi}}$$

$$\begin{aligned} \tilde{g}_{99} &= \frac{1}{g_{99}}, \qquad \tilde{g}_{9\mu} = \frac{B_{9\mu}}{g_{99}}, \qquad \tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{B_{9\mu}B_{9\nu} - g_{9\mu}g_{9\nu}}{g_{99}}, \\ \tilde{B}_{9\mu} &= -\tilde{B}_{\mu 9} = \frac{g_{9\mu}}{g_{99}}, \qquad \tilde{B}_{\mu\nu} = B_{\mu\nu} + \frac{g_{9\mu}B_{9\nu} - B_{9\mu}g_{9\nu}}{g_{99}}, \\ e^{\tilde{\Phi}} &= \sqrt{g_{99}}\, e^{\Phi} \end{aligned}$$

$$F_{M N_1 \cdots N_p} \overset{\mathrm{T_M}}{\longleftrightarrow} F_{N_1 \cdots N_p}$$

"Buscher rules"

Example: T^3 with H_3

T^3 → $(x,y,z) \sim (x+1,y,z) \sim (x,y+1,z) \sim (x,y,z+1)$

$\int_{T^3} H_3 = N$ $\quad$ $B_{xy} = Nz$

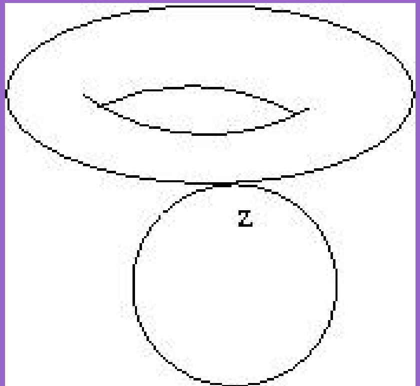
T- dualize along x $\xrightarrow{\text{Buscher}}$

$$\begin{aligned} ds^2 &= (dx - Nz)^2 + dy + dz^2 = |dx + \tau dy|^2 + dz^2, \\ B &= 0 \end{aligned}$$

Twisted torus $\quad H_{xyz} \overset{\mathrm{T}_x}{\longleftrightarrow} \omega^x_{yz} = N \quad\quad \tau = Nz + i$

$$(x,y,z) \sim (x+1,y,z) \sim (x,y+1,z) \sim (x+\omega^x_{yz}y, y, z+1)$$

$z \to z+1$ → $\tau(z) \to \tau(z) + \omega^x_{yz} y$



T- dualize along y $\omega^{x}_{zy} \overset{T_n}{\longleftrightarrow} Q^{xy}_{z}$ Non geometric

Buscher

$$ds^2 = \frac{1}{1+(Nz)^2}(dx^2+dy^2)+dz^2$$

$$B_{xy} = \frac{Nz}{1+(Nz)^2}$$

mix under $z \rightarrow z+1$

Well defined solution of 10D SUGRA

$$H_{mnp} \overset{T_m}{\longleftrightarrow} -\omega^{m}_{np} \overset{T_n}{\longleftrightarrow} -Q^{mn}_{p} \overset{T_p}{\longleftrightarrow} R^{mnp} \ .$$

What is the 10D interpretation of fluxes?

F-Theory?

•F-theory (p,q) 7-branes transverse to CP1 coupling electrically to a non-linear doublet (triplet + constraint) of 8-forms

[Bergshoeff et al.]

$$\int -C_8 \wedge QF_3 + \tilde{C}_8 \wedge PH_3 + C_8' \wedge (QH_3 + PF_3)$$

•New fluxes can be understood as non geometric compactifications

patching with diffeomorphisms, T^2 and ST^2S dualities

Axio-dilaton is not monodromy neutral

→ **U-folds**

NON-GEOMETRIC BACKGROUND

A geometric string solution has background fields in overlapping coordinate patches related by diffeomorphisms and gauge transformations, while for a non-geometric background this is generalised to allow transition functions involving duality transformations.

[Frey, Grana]
[Dall'Agata]
[Marchesano, Schulgin]
[Hull]
....

T-folds   Diff+ T-dualities

U-fold Diff +T+ S dualities

D=4 Is not just the reduction of D=10 SUGRA effective theory

F-Theory:

Type IIB comapctification where the

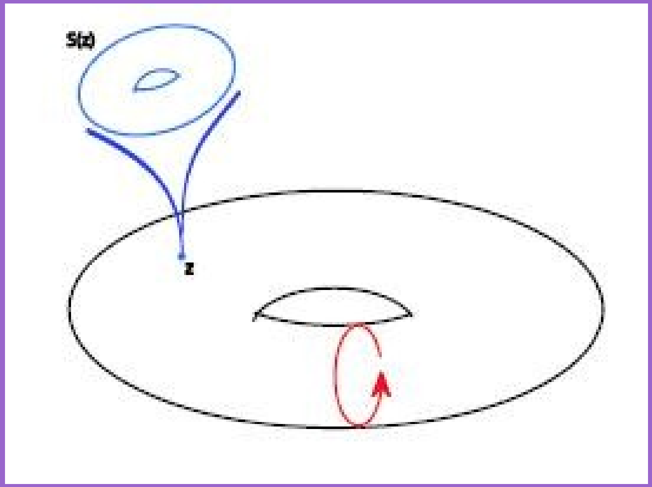
axio-dilaton $S = e^{-\phi} + iC_0$ Is the complex structure of an elliptic fiber T^2

Example: $K_3 = T^2$ **over** CP^1

$$y^2 = x^3 + P(z)x + Q(z) \ ,$$

$P^3 + Q^2$ order 24 polinomyal

z Coordinate on the base (sphere)

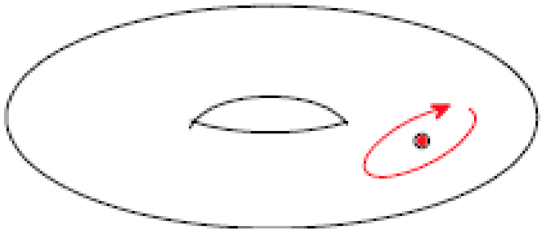


Invariance under global monodromies $S(\Lambda z) = S(z), \qquad \Lambda \in SL(2, \mathbb{Z})$

$$S(z) = -ij^{-1}\left(\frac{P^3(z)}{P^3(z)+Q^2(z)}\right)$$

F-theory on K3 x T^2 x T^2

Local monodromies around the singularities:



$$\mathcal{N}_7 \equiv \begin{pmatrix} \frac{pq}{2} & p^2 \\ -q^2 & -\frac{pq}{2} \end{pmatrix}$$

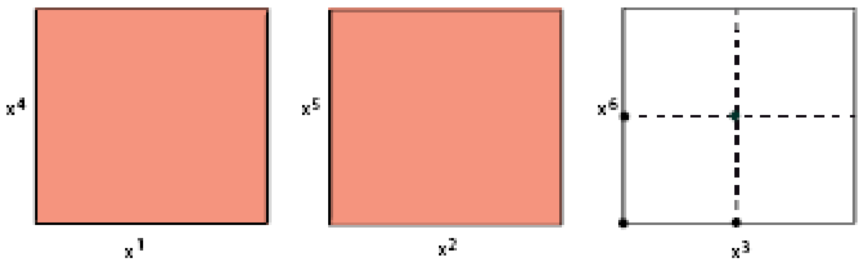
$$S \to e^{\mathcal{N}_7} S$$

(p,q) 7-branes transverse to CP^1 coupling electrically to a non-linear doublet (triplet + constraint) of 8-forms [Bergshoeff et al.]

Moduli space of 4d theory: positions of 7-branes + S, U_i, T_i

$$\underbrace{\text{positions of 7-branes} + S,\ U_i,\ T_i}_{SL(2,\mathbb{Z})^7 \subset SO(6,6) \times SL(2,\mathbb{Z})}$$

Orientifold limit: $P^3(z) \propto Q^2(z)$. Then $S \to$ const. and the CP^1 degenerates into $T^2/\mathbb{Z}_2$



4 D7-branes + 1 O7-plane per fixed point

Non-geometric F-theory compactification

- Acting with those symmetries we can build new classes of F-theory compactifications

solutions interpolating between different orientifold involutions

[Frey, Grana]
[Dall'Agata]
[Marchesano,Schulgin]

Example:

$$ds^2 = Z^{-1/2} ds^2_{\mathbb{R}^{1,3}} + Z^{1/2} \left[\sum_{k=1,2} \frac{t_3 e^{\phi} t_k}{\text{Re } U} |dx^k + iU dx^{k+3}|^2 + \frac{Z^{1/2}}{g_s t_3} |dx^3 + iT_3 dx^6|^2 \right] ,$$

$$S = t_1 t_2 t_3 + \frac{N^2}{8\pi} \log\, \vartheta_1(2z^3, -iT_3) ,$$

$$C_2 = N x^3 (dx^1 \wedge dx^5 - dx^4 \wedge dx^2) ,$$

$$B_2 = \frac{N t_3 x^6}{t_1 t_2 t_3 + \frac{N^2}{8\pi} \log\, |\vartheta_1(2z^3, -iT_3)|^2} (dx^1 \wedge dx^5 - dx^4 \wedge dx^2) ,$$

$$F_5 = C_2 \wedge H_3 + F_3 \wedge B_2 - \frac{1}{2} F_1 \wedge B_2 \wedge B_2 - C_0 \wedge H_3 \wedge B_2 ,$$

S is still holomorphic but not monodromy neutral → patching with diffeomorphisms, T^2 and ST^2S dualities. (U-fold)

Globally non-geometric compactifications

- **How do fluxes manifest in 4D ?**

D=4 vector multiplets

- **Kaluza Klein**, from dimensional reduction of metric and p-forms
- **7-branes** ADE symmetries
- **3-branes** gauge symmetries

10 D fluxes gauge KK vectors

$$A_\mu \equiv B_{\mu p} X^p \qquad \delta A_{\mu c} = \partial_\mu \lambda_c - Q_c^{ab} \lambda_a A_{\mu b}$$

$$[X^a, X^b] = Q_p^{ab} X^p \quad \Longrightarrow \quad Q_p^{[ab} Q_t^{c]p} = 0$$

By systematic application of SL(2,Z) transformations, the

full KK gauge algebra can be found

Kaluza-Klein vectors

$$
\begin{aligned}
[X^a, X^b] &= -\ \tilde{F}^{abp} Z_p + Q_p^{ab} X^p \ , \\
[X^a, Z_b] &= -Q_b^{ap} Z_p \ , \\
[X^a, \bar{X}^b] &= Q_p^{ab} \bar{X}^p -\ \tilde{F}^{abp} \bar{Z}_p \ , \\
[\bar{X}^a, Z_b] &= [X^a, \bar{Z}_b] = -Q_b^{ap} \bar{Z}_p \ , \\
[\bar{X}^a, \bar{X}^b] &= P_p^{ab} \bar{X}^p - \tilde{H}^{abp} \bar{Z}_p \ , \\
[\bar{X}^a, \bar{Z}_b] &= -P_b^{ap} \bar{Z}^p \ , \\
[\bar{Z}_a, \bar{Z}_b] &= [Z_a, Z_b] = [Z_a, \bar{Z}_b] = 0 \ .
\end{aligned}
$$

In 4d S-duality manifests as electric-magnetic duality $(X \leftrightarrow \bar{X})$

The following information is encoded in the Jacobi identities of the algebra:

Bianchi identities

$$
\begin{aligned}
Q_p^{[ab} Q_l^{c]p} &= 0 \ , \\
P_p^{[ab} P_l^{c]p} &= 0 \ , \\
Q_p^{[ab} P_l^{c]p} = P_p^{[ab} Q_l^{c]p} &= 0 \ ,
\end{aligned}
$$

Tadpole cancellation

$$
\begin{aligned}
Q_p^{[ab} \tilde{F}^{c]lp} + \tilde{F}^{p[ab} Q_p^{c]l} &= 0 \quad \Leftrightarrow \quad QF_3 = 0 \ , \\
\tilde{H}^{p[ab} P_p^{c]l} + P_p^{[ab} \tilde{H}^{c]lp} &= 0 \quad \Leftrightarrow \quad PH_3 = 0 \ , \\
Q_p^{l[a} \tilde{H}^{bc]p} - P_p^{[ab} \tilde{F}^{c]lp} = P_p^{l[a} \tilde{F}^{bc]p} - Q_p^{[ab} \tilde{H}^{c]lp} &= 0 \quad \Leftrightarrow \quad \begin{cases} PF_3 + QH_3 = 0 \\ Q_{[b}^{ap} H_{cd]p} - P_{[b}^{ap} F_{cd]p} = 0 \end{cases} ,
\end{aligned}
$$

Gaugings and branes

U(1) generators can be easily incorporated in the algebra

i.e. Magnetized D9-branes

$$n_i \, \mathcal{F}_{i,i+3} \equiv n_i \int_{[\omega_i]} F = m_{i+3}$$

Chern Simons -couplings

$$c_0 = -n\mathcal{F}_{14}\mathcal{F}_{25}\mathcal{F}_{36} = -m_4 m_5 m_6 \,,$$

$$c_1 = n\mathcal{F}_{14} = m_4 n_2 n_3 \,,$$

$$c_2 = n\mathcal{F}_{25} = m_5 n_1 n_3 \,,$$

$$c_3 = n\mathcal{F}_{36} = m_6 n_1 n_2 \,.$$

$$c_i \int \partial_\mu a^i A^\mu \,, \qquad c_0 \int \partial_\mu a^0 A^\mu$$

with

$$da^i = *_4 dB_2^i \,, \qquad da^0 = - *_4 dC_2 \,, \qquad B_2^i = \int_{[\tilde{\omega}_i]} C_6 \,.$$

a^i **(RR axions S,T,U) get shifted under U(1)**

$$a^i \to a^i + c_i \chi$$

Green-Schwarz mechanism

Gaugings and branes

Freed-Witten anomaly cancellation: the flux background should not spoil the shift symmetries appearing in the Green-Schwarz mechanism

[Camara, Font, Ibanez]

Superpotential $$W_{O9} = \int_{\mathrm{T}^6} \Omega \wedge (F_3 + \omega J_c) \ ,$$

must be invariant under axionic shifts $$J_c \to J_c + n\mathcal{F}\chi$$

$$\omega^p_{[ab} n \mathcal{F}^I_{c]p} = 0$$

We propose

$$\begin{aligned} [Z_a, Z_b] &= \omega^p_{ab} Z_p + n\mathcal{F}^I_{ab} X^I - \boldsymbol{F}_{abp} Y^p \ , \\ [Z_a, Y^p] &= -\omega^p_{aq} Y^q \ , \\ [Z_a, X^I] &= \mathcal{F}^I_{aq} Y^q \ , \end{aligned}$$

$$\begin{aligned} \omega^p_{[ab}\omega^l_{c]p} &= 0 \ , \\ \omega^p_{[ab} n\mathcal{F}^I_{c]p} &= 0 \ , \\ \boldsymbol{F}_{p[ab}\omega^p_{c]l} + \omega^p_{[ab}\boldsymbol{F}_{c]lp} &= -n \sum_I \mathcal{F}^I_{[ab}\mathcal{F}^I_{c]l} \ . \end{aligned}$$

Generalize to (p,q) 7-branes in F-theory set up

- Restrictions on fluxes can be found:

Bianchi Identities, Tadpole cancellation, Anomaly Freed-Witten free

- *Bianchi identities:*

$$Q_p^{[ab}Q_l^{c]p} = 0$$

$$P_p^{[ab}P_l^{c]p} = 0 \ ,$$

$$Q_p^{[ab}P_l^{c]p} = P_p^{[ab}Q_l^{c]p} = 0 \ ,$$

$$Q_p^{l[a}\tilde{H}^{bc]p} + Q_p^{[ab}\tilde{H}^{c]lp} - P_p^{l[a}\tilde{F}^{bc]p} - P_p^{[ab}\tilde{F}^{c]lp} = 0 \ ,$$

- *7-brane tadpoles:*

$$(QF_3)_k = -\sum_I (p_k^I)^2 d_k^I$$

$$(PH_3)_k = -\sum_I (q_k^I)^2 d_k^I \ ,$$

$$(PF_3 + QH_3)_k = -2\sum_I p_k^I q_k^I d_k^I$$

- *Freed-Witten anomalies:*

$$Q_p^{[ab}(c_I^k)^{c]p} = 0 \ ,$$

$$P_p^{[ab}(c_I^k)^{c]p} = 0 \ ,$$

Comparison with gauged sugra

- **Gaugings encoded in tensors:** $f_{\alpha MNP}$, $\xi_{\alpha M}$ [Schon & Weidner]

projection $SL(2,\mathbb{Z}) \times SO(6,6+n;\mathbb{Z}) \Rightarrow SL(2,\mathbb{Z})^7$ **leaves**

$$f_{\alpha MNP}, \qquad (M \bmod 3) \neq (N \bmod 3) \neq (P \bmod 3)$$

- **Algebra and Jacobi identities:**

$$\begin{aligned} [Z_{\alpha A}, Z_{\beta B}] &= \delta^{\rho}_{\beta} f_{\alpha AB}{}^{P} Z_{\rho P} \\ f^{P}_{\alpha[AB} f^{L}_{\beta B]P} &= 0 \\ f^{P}_{+AB} f^{L}_{-BP} - f^{P}_{-AB} f^{L}_{+BP} &= 0 \end{aligned}$$

Complete agreement with our results if we identify

$$F_{abc} = -f_{+abc}$$

$$Q_a^{bc} = \tilde{\eta}_a{}^P \hat{\eta}^{bQ} \hat{\eta}^{cR} f_{+PQR} \ ,$$

$$H_{abc} = -f_{-abc} \ ,$$

$$P_a^{bc} = \tilde{\eta}_a{}^P \hat{\eta}^{bQ} \hat{\eta}^{cR} f_{-PQR}$$

$$(c_I)^{ab} = \frac{1}{p^I} \hat{\eta}^{aQ} \hat{\eta}^{bR} f_{+(12+I)QR} = \frac{1}{q^I} \hat{\eta}^{aQ} \hat{\eta}^{bR} f_{-(12+I)QR} \ ,$$

$$\tilde{\eta}_M{}^N = \begin{pmatrix} 0 & 0 & 0 & \mathbb{I}_3 \\ 0 & 0 & -\mathbb{I}_3 & 0 \\ 0 & \mathbb{I}_3 & 0 & 0 \\ -\mathbb{I}_3 & 0 & 0 & 0 \end{pmatrix} , \quad \hat{\eta}^{MN} = -\hat{\eta}_{MN} = \begin{pmatrix} 0 & \mathbb{I}_3 & 0 & 0 \\ -\mathbb{I}_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbb{I}_3 \\ 0 & 0 & -\mathbb{I}_3 & 0 \end{pmatrix} .$$

D=10 Fluxes -Sugra structure constants dictionary

Comparison with gauged sugra

Moreover, requiring the Scalar potential

$$V_{\mathcal{N}=4} = -\frac{1}{16}\Big[f_{\alpha MNP}f_{\beta QRS}M^{\alpha\beta}\left(\frac{1}{3}M^{MQ}M^{NR}M^{PS} + \left(\frac{2}{3}\eta^{MQ} - M^{MQ}\right)\eta^{NR}\eta^{PS}\right)$$
$$-\frac{4}{9}f_{\alpha MNP}f_{\beta QRS}\epsilon^{\alpha\beta}M^{MNPQRS} + 3\xi^M_\alpha\xi^N_\beta M^{\alpha\beta}M_{MN}\Big]$$

to have a **N=1** structure

$$V_{\mathcal{N}=1} = e^K\left(\sum_{\mathbb{T}_i}(\mathbb{T}_i+\bar{\mathbb{T}}_i)^2|D_{\mathbb{T}_i}W|^2 - 3|W|^2\right) ,$$

Leads to same constraints plus

$$\tilde{F}^{pqr}H_{pqr} = 0$$
$$H_{pq(a}Q^{pq}_{b)} - F_{pq(a}P^{pq}_{b)} = 0 , \qquad \hat{\eta}_{pq}P^{p[a}_{[c}Q^{b]q}_{d]} = 0 .$$

Conclusion:

- Duality requires existence of many geometrical and non geometrical fluxes
- F-theory with a non-neutral dilaton provides an interpretation. U-folds
- Gauge algebra can be found for KK and D7 brane vectors
- Jacobi identities encode global 10 D constraints on fluxes: BI, F-theory 7-branes tadpoles, FW anomalies
- N=1 constraints
- Direct connection with D=4 gauged supergravity
- D=4 gauged supergravity incorporates stringy information

Outlook

- Extend to primed fluxes
- Understand N=1 constraints
- Solutions with stabilized moduli, phenomenology?
- Understand non geometric fluxes from Generalized Geometry
- ....

C. Hull
M.Graña, R.Minasian,M.Petrini,D.Waldram
....

Generalized Geometry

$$T \oplus {}^*T$$

$$\mathcal{L} = -\frac{1}{2}\mathcal{H}_{IJ}(Y)\eta^{\alpha\beta}\partial_\alpha \mathbb{X}^I \partial_\beta \mathbb{X}^J - \eta^{\alpha\beta}\mathcal{J}_{IA}(Y)\partial_\alpha \mathbb{X}^I \partial_\beta Y^A + \mathcal{L}_N(Y)$$

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix},$$

Generalized metric

$$\mathbb{X}^I = (X^i, \tilde{X})$$

Transition functions contain T-dualities

Include RR fluxes
U dualities

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Type IIA orientifold (O6): $\otimes_{j=1}^{3} T_j^2/\Omega_P(-1)^{F_L}\sigma$

$$\sigma_A(x^i) = x^i \quad ; \quad \sigma_A(y^i) = -y^i \quad ; \quad \sigma_A(\Omega) = \Omega^* \quad ; \quad \sigma(J) = -J$$

In terms of complexif ed forms

$$J_c = B + iJ \quad ; \quad \Omega_c = C_3 + i\mathrm{Re}(C\Omega)$$

with

$$C = e^{-\phi_4}\left[\frac{1}{8i}\int \Omega \wedge \Omega^*\right]^{-1/2} \quad ; \quad e^{\phi_4} = e^{\phi}/(Vol)^{1/2}$$

CLOSED MODULI

	dim.	form	dual form	moduli
Kähler	$h^-_{(1,1)}$	ω_A	$\widetilde{\omega}_A$	$T_A = -i\int J_c \wedge \widetilde{\omega}_A$
c.s + dilaton	$1 + h_{(1,2)}$	α_L	β_L	$U_L = i\int \Omega_c \wedge \beta_L$

Type IIA fluxes

Turn on RR and NSNS fluxes

$$\overline{F}_0 = -m \quad ; \quad \overline{F}_2 = \sum_{A=1}^{h_{11}^-} q_A \omega_A \quad ; \quad \overline{F}_4 \quad = \quad \sum_{A=1}^{h_{11}^-} e_A \widetilde{\omega}_A \quad ; \quad \overline{F}_6 = e_0 dVol_6$$

$$\overline{H}_3 = \sum_{L=0}^{h_{12}} h_L \beta_L$$

Geometric fluxes (Scherk-Shcwarz)

$$ds^2 = \sum_k (dx^k + \omega^k_{pq} x^q dx^p)^2$$

Correspond to $d\eta^p = -\frac{1}{2}\omega^p_{mn}\eta^m \wedge \eta^n.$

EXAMPLE: $\mathrm{T}^6 = \otimes_{i=1}^3 \mathrm{T}_i^2$

3-forms with one leg on each sub-torus

$$\alpha_0 = dx^1 \wedge dx^2 \wedge dx^3 \quad ; \quad \beta_0 = dy^1 \wedge dy^2 \wedge dy^3 \ ,$$
$$\alpha_1 = dx^1 \wedge dy^2 \wedge dy^3 \quad ; \quad \beta_1 = dy^1 \wedge dx^2 \wedge dx^3 \ , \ldots$$

Closed 2-forms and their dual 4-forms

$$\omega_i = -dx^i \wedge dy^i \quad ; \quad \widetilde{\omega}_i = dx^j \wedge dy^j \wedge dx^k \wedge dy^k \quad ; \ i \neq j \neq k \ .$$

Complex structure parameters $\tau_j = \frac{1}{e_{jx}^2}(A_j + i\, e_{jx} \cdot e_{jy})$ where $e_{jx}\ (e_{jy})$

Are lattice vectors of sizes $R_x^j\ (\ R_y^j)$ and $(2\pi)^2 A_j$ area of T_j^2

Kaler and holomorphic 3-foms

$$J = \sum_{i=1}^{3} A_i \omega_i$$
$$\Omega = (dx^1 + i\tau_1\, dy^1) \wedge (dx^2 + i\tau_2\, dy^2) \wedge (dx^3 + i\tau_3\, dy^3)$$

Scherk-Schwarz

$$X^M \quad \Rightarrow \quad \begin{array}{ll} x^\mu & \mu = 1, \ldots d(4) \\ y^a & a = 1, \ldots d_I(6) \end{array}$$

$$ds^2 = g_{\mu\nu}(x) dx^\mu \otimes dx^\nu + g_{ab}(x)(dy^a + A^a_\mu(x) dx^\mu) \otimes (dy^b + A^b_\mu(x) dx^\nu)$$

$$g_{MN} \quad \to \quad g_{\mu\nu}(x); \quad g_{ab}(x); \quad A^a_\mu(x)$$

$$\Downarrow$$

$$g_{MN} \quad \to \quad V^n_M \qquad V^n_M \quad \to \quad V^m_\mu; \quad \Phi^i_a(x); \quad A^a_\mu(x)$$

$$\delta V^m_M \quad = \quad \xi^P \partial_P V^m_M + \partial_M \xi^P V^m_P$$

choose $\xi^P(x, y) = \xi^P(x)$

$$\delta V^m_\mu \quad = \quad \delta \Phi^i_a = 0$$

$$\delta A^a_\mu \quad = \quad \partial_\mu \xi^a$$

$$U(1)^6$$

$$\xi^\mu(x,y) = \xi^\mu(x)$$
$$\xi^a(x,y) = [U^{-1}]^a_b\,\xi^b(x)$$

(Scherk-Schwarz 1979)

$$\delta V^m_\mu = 0$$
$$\delta \Phi^i_a = \omega^a_{bc}\,\Phi^i_c$$
$$\delta A^a_\mu = \partial_\mu \xi^a + \omega^a_{bc}\,\xi^b\,A^c_\mu$$

$$\omega^c_{ab} = [U^{-1}]^{a'}_a\,[U^{-1}]^{b'}_b\,(\partial_{b'} U^c_{a'} - \partial_{a'} U^c_{b'})$$

Metric fluxes

⇩

gaugings

$$\omega^c_{ab}$$