Chern-Simons Branes and the Green-Schwarz Superstring Action

New Trends in Quantum Gravity II
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Introduction

- ## Quantum Field Theory

  Surprises: Renormalization, Spontaneous Symmetry Breaking, Anomalies

  Lessons: Gauge invariance as guiding principle

- ## Theories of Extended Objects

  From Dirac, Nambu, Susskind, Nielsen, 1960´s and 1970´s to Superstring Theory

- ## Quantum Gravity

  Perturbative vs. Non perturbative, background dependent vs. background independent

Chern-Simons Gravity

- Background independent gauge theories for space-time groups and supergroups
- CS gravity in 2+1 equivalent to GR (Van Nieuwenhuizen, Achúcarro & Townsend, Witten,...). Formulation as a gauge theory makes quantization possible. BTZ black hole solutions.
- In higher odd dimensions CS gravity has a rich dynamical structure including black hole solutions (Chamsedine, Bañados, Bunster, Troncoso, Zanelli,...).
- Under certain conditions there exist dimensional reduction procedures leading to extensions of 4D GR.

CHERN-SIMONS GAUGE THEORIES

$A = A^a_\mu dx^\mu T^a \Leftarrow$ gauge potential

$\downarrow$ Generators of group G

$F = dA + A^2 \Leftarrow$ Field strength/Curvature

$\langle T^{a_1} \cdots T^{a_{m+1}} \rangle = g^{a_1 \cdots a_{m+1}}$

$\Uparrow$

symmetric invariant tensor in the Lie algebra

Lagrangian $\overset{?}{=} \langle F \wedge \cdots \wedge F \rangle = \langle F^{m+1} \rangle$

$\Uparrow$

Doesn't work $\langle F^{m+1} \rangle = d\,Q_{2m+1}(A, F)$

$Q_{2m+1}(A, F) = (m+1)\int_0^1 dt \langle A F_t^m \rangle$

$\Uparrow$

Chern-Simons form, $A_t = tA$, $F_t = dA_t + A_t^2$

Chern-Simons Action: $\boxed{I = \int_M Q_d}$

Field Equations:

$\langle F^m T^a \rangle = 0$ $\qquad$ $d = 2m+1$

Gauge Transformations:

$\delta_\lambda Q_{2m+1} = dB_{2m} \Leftarrow$ Quasi-invariant

$\delta I =$ Boundary contribution

Chern-Simons Gravity

(4)

Spacetime gauge group $\begin{cases} dS \\ AdS \\ \text{Poincaré} \end{cases}$

independent Variables
spin connection
$\omega^{ab} = \omega^{ab}_{M} dx^{M}$
vielbein
$e^{a} = e^{a}_{M} dx^{M}$

AdS : $SO(d-1,2)$, $d = 2m+1$

Generators : J_{AB} $A,B = 0, \dots, d$

Algebra : $[J_{AB}, J_{CD}] = \eta_{BC} J_{AD} - \eta_{AC} J_{BD} + \eta_{AD} J_{BC} - \eta_{BD} J_{AC}$

"Splitting" in "Lorentz" and "translations"

"Lorentz": J_{ab} , $a,b = 0, \dots, d-1$

"translations": $P_a = J_{a,d}$

Invariant tensor (symmetric)

$$\langle J_{A_1 A_2} \cdots J_{A_{d-1} A_d} \rangle = \frac{k\, 2^m}{(m+1)} \epsilon_{A_1 \cdots A_d}$$

Gauge Potential
$$A = \tfrac{1}{2} \omega^{ab} J_{ab} + \frac{e^a}{\ell} P_a$$

$$F = dA + A^2 = \tfrac{1}{2}\left(R^{ab} + \frac{e^a e^b}{\ell^2}\right) J_{ab} + \frac{T^a}{\ell} P_a$$

ℓ : AdS "radius". We will set $\ell = 1$; can be reintroduced by dimensional analysis

Chern-Simons gravity action

$$I = k \int_M \int_0^1 dt\, \epsilon_{a_1 \cdots a_{2m+1}} R_t^{a_1 a_2} \wedge \cdots \wedge R_t^{a_{2m-1} a_{2m}} \wedge e^{a_{2m+1}}$$

$$R_t^{ab} = R^{ab} + t^2 e^a \wedge e^b$$

Note: $T^a = de^a + \omega^a{}_c e^c$, $R^{ab} = d\omega^{ab} + \omega^a{}_c \omega^{cb}$

TRANSGRESSIONS

$$\boxed{\mathcal{T}_{2n+1}(A, F) = (n+1)\int_0^1 dt \, \langle (A-\bar{A}) F_t^{\,n} \rangle}$$

$A, \bar{A}$: two sets of gauge fields

$\Delta A = A - \bar{A} \qquad A_t = tA + (1-t)\bar{A}$

$F_t = dA_t + A_t^2$

· It can be shown that

$$\boxed{\mathcal{T}_{2n+1}(A, \bar{A}) = Q_{2n+1}(A) - Q_{2n+1}(\bar{A}) - dB_{2n}(A, \bar{A})}$$

$B_{2n} = -n(n+1)\int_0^1 ds \int_0^1 dt \, \langle A_t \Delta A \, F_{st}^{\,n-1} \rangle$

$F_{st} = s F_t + s(s-1) A_t^2$

Transgressions are gauge invariant not quasi-invariant

Transgressions as lagrangians: generalization of CS actions, gauge invariant

Interpretations of the doubling of fields

I. A & $\bar{A}$ both dynamical

I.a. A & $\bar{A}$ live in the same manifold

I.b. A & $\bar{A}$ live in M & $\bar{M}$ with $\partial M \equiv \partial \bar{M}$

II. A dynamical, $\bar{A}$ background

From Heterotic to Chern-Simons branes

- Heterotic branes: coupling of extended objects to gauge fields in a standard supergravity backgound (Dixon, Duff & Sezgin 1992), generalization of Heterotic Strings to higher (even) space-time dimensions.
- M. Green on the relationship between Green-Schwarz Superstring action and Chern Simons action.
- Problem: construction of actions for extended objects for CS gravity and supergravity.

HETEROTIC BRANES AND DIXON-DUFF-SEZGIN (DDS) ACTION

Coupling of extended objects to gauge fields
d: dimensione of the brane world-volume
D: dimension of the space-time

$$S_{DDS} = \underbrace{S_d^K}_{\text{kinetic}} + \underbrace{S_d^{WZW}}_{\text{Wess-Zumino-Witten}}$$

$$S_d^K = \int d^d\xi \left\{ -\tfrac{1}{2}\sqrt{-\gamma}\,\gamma^{ij}\left[\partial_i X^r \partial_j X^s g_{rs}(X^p) + J_i^a J_j^a\right] + \tfrac{1}{2}(d-2)\sqrt{-\gamma} \right\}$$

ξ^i : world volume coordinats $\quad i,j,k = 0,\dots,d-1$
$X^r(\xi)$: space-time coordinates $\quad r,s,p = 0,\dots,D-1$
$g_{rs}(x)$: metric of the background space-time

$$J_i^a = \partial_i X^r A_r^a - \partial_i Y^\ell K_\ell^a$$

$A_r^a(x)$: gauge potentials for gauge group G with generators T^a
$Y^\ell(\xi)$: coordinates in the group manifold G
$K_\ell^a(Y)$: Maurer-Cartan (MC) left invariant forms

$$K_\ell = K_\ell^a T^a = g^{-1}(Y)\frac{\partial}{\partial Y^\ell} g(Y)$$

As differential forms

$$A = A_r^a T^a dx^r \quad \& \quad K = K_\ell^a T^a dy^\ell$$

pull-backs

$$A = A_r^a T^a \partial_i X^r d\xi^i \quad \& \quad K = K_\ell^a T^a \partial_i Y^\ell d\xi^i$$

Heterotic BRANES (cont.)

$J \equiv A - K$ $\quad\|\quad$ $\underset{\overset{\Uparrow}{\text{MC equation}}}{dK + K^2 = 0}$

$\downarrow$
not a current

$$S_d^{WZW} = \int \{ B_d + C_d - b_d \} = \int \mathcal{B}_d$$

$$B_d = \frac{1}{d!} B_{r_1 \cdots r_d} \partial_{i_1} X^{r_1} \cdots \partial_{i_d} X^{r_d} \, dJ^{i_1} \cdots dJ^{i_d}$$

$\Uparrow$
RR-field

$$b_d = \frac{1}{d!} b_{r_1 \cdots r_d} \partial_{i_1} X^{r_1} \cdots \partial_{i_d} X^{r_d} \, dJ^{i_1} \cdots dJ^{i_d}$$

$\Uparrow$
WZW form satisfying $db_d = -Q_{d+1}(K, 0)$

$$dQ_{d+1}(K,0) = \langle \underset{\overset{\shortparallel}{0}}{F}{}^{\frac{d+2}{2}} \rangle = 0$$

The d-form $C_d(A, K, F)$ is the boundary term of the transgression, with
$A \leftrightarrow A \qquad \bar{A} \leftrightarrow K$

Gauge invariance of DDS action

$\underline{\delta_\lambda B_d = + Q_d^1(A, F, \lambda)}$ where $\delta_\lambda Q_{d+1}(A, F) = - d Q_d^1(A, F, \lambda)$

$\Rightarrow \underline{\delta_\lambda b_d = Q_d^1(K, 0, \lambda)}$ $\qquad$ $\underline{H_{d+1} = dB_d + Q_{d+1}(A, F)}$

gauge invariant field strength

$\delta_\lambda C(A, K, F) = - Q_d^1(A, F, \lambda) + Q_d^1(K, 0, \lambda) \Leftarrow$ from transgression

$$\Rightarrow \delta_\lambda \mathcal{B}_d = 0 \Rightarrow \boxed{\delta_\lambda S_d^{WZW} = 0}$$

HETEROTIC BRANES (CONT.)

Gauge invariance of DDS action (cont.)

J^k_j invariant because $J^a_i J^a_j = \langle J_i \cdot J_j \rangle$ invariante, as $J \to g^{-1} J g$ (covariant)

Further invariances of DDS action

- General coordinate transformations of the background
- General coordinate transformations of the world volume
- If the background is flat $g_{rs}(x) = \eta_{rs} \Rightarrow$ invariant under global Poincaré transformations

HETEROTIC BRANES (CONT.)

Gauge invariance of DDS action (cont.)

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CHERN-SIMONS BRANES

IDEAS:

1. Going from a gauge group G to a space-time group
2. Going from a suphrgravity background $g_{rs}(x)$ to a CS gravity or SUGRA background
3. Instead of a pure gauge potentials K, two dynamical potentials A_0 & A_1
4. Instead of even dimension d, CS branes live in an odd dimensional space-time. Can be regarded as the boundary of a CS Brane in the case that boundary is restricted to a d-1 submanifold & A_0 is a pure gauge potential K
5. Action defined as

$$\boxed{S = \sum_{m=0}^{N} \alpha_m \int_{S^{2m+1}} \mathcal{Y}_{2m+1}(A_1, A_0)}$$

CHERN-SIMONS-BRANES (cont.)

$$S = \sum_{m=0}^{N} \alpha_m \int_{S^{2m+1}} \mathcal{Z}_{2m+1}(A_1, A_0)$$

$S^{2N+1} \Leftarrow$ space-time manifold

$S^{2m+1} \Leftarrow$ world-volumes of the branes

$S^{2m+1} \subset S^{2N+1}$

$\partial S^{2m+1} = \Omega^{2m} \Leftarrow$ Branes with boundaries

Dynamical variables

→ Gauge potentials $A^I_m(x)$ $\quad I = 1, 0$, $\; m = 0, \dots, 2N$

→ Embedding coordinates $X^m_{(2m+1)}(x^i_{(2m+1)})$

with $x^i_{(2m+1)}$ local coordinates in S^{2m+1}

& $\xi^{ij}_{(2m)}$ local coordinates in Ω^{2m}

Kinetic Terms (Optional)

$$S_K^{(2m)} = \frac{1}{2} \int_{\Omega^{2m}} d^{2m} \xi_{(2m)} \sqrt{-\gamma_{(2m)}} \left\{ \gamma^{ij}_{(2m)} \langle J_i J_j \rangle - 2(m-1) \right\}$$

auxiliary world-volume metric

Also Born-Infeld like forms are possible

CHERN-SIMONS BRANES (cont.)

Invariances of the action

→ General coordinate transformations

→ Gauge transformations

Equations of Motion

For CS theory without branes

$\boxed{\langle F^n T^a \rangle = 0}$ ⇐ $F=0$ is a solution but <u>not</u> the only one ⇒

⇒ rich dynamical structure

For a theory with CS branes

$\boxed{\frac{\delta S}{\delta A_r} = 0}$ gives $\boxed{J^{(r)\,mI} = 0}$ ⟺

⟺ $\boxed{\langle F^n T^a \rangle = \sum \left\{ \begin{array}{l} \text{distributional} \\ \text{currents carried} \\ \text{by the branes} \end{array} \right\}}$

$\frac{\delta S}{\delta X} = 0$ ⇒ (if there is no kinetic term)

$$\boxed{\langle F^n \rangle_{\alpha \mu_1 \cdots \mu_n} \frac{\partial X^{\mu_1}}{\partial \sigma^{i_1}} \cdots\cdots \frac{\partial X^{\mu_{2k+1}}}{\partial \sigma^{i_{2k+1}}} \epsilon^{i_1 \cdots\cdots i_{2k+1}} = 0}$$

<u>Heuristically</u>: the extended objects are sources of gauge fields and their motion is affected by those fields

CHERN-SIMONS BRANES (cont.)

Quantization of the coefficients in the action

$\leftarrow \Omega^{p+2}$ — Branes without boundaries

$\leftarrow S^{p+1}$ world-volume of the brane

$$S = k \int_{S^{p+1}} Q_{p+1} = k \int_{\Omega^{p+2}} \langle F^{\frac{p}{2}+1} \rangle$$

If S^{p+1} shrinks to zero $e^{i\frac{S}{k}} \longrightarrow 1 \Longrightarrow$

$S = 2\pi m \Longrightarrow \underbrace{k \int_{\Omega^{p+2}} \langle F^{\frac{p}{2}+1} \rangle} = 2\pi m \hbar$

m integer

if $\int_{\Omega^{p+2}} \langle F^{\frac{p}{2}+1} \rangle =$ integer (because index theorem, etc.)

$\Rightarrow$ k is quantized $\rightarrow$ there is a non-trivial flux through $\Omega^{2n} \Longrightarrow$

$\Rightarrow$ $(d-p-4)$-brane $\longleftrightarrow$ "solitonic" or "magnetic" dual brane.

For instance in 11D there would be a solitonic 5-brane corresponding to a fundamental CS 2-brane

Transgressions and CS gravity

Boundary terms for every odd dimension providing a built in regularization that give (Olea, Troncoso, Zanelli, PM**):**

- Finite Noether conserved charges
- Finite black hole thermodynamics
- Well defined action principle
- The boundary term provided by Transgressions is “portable” to odd dimensional AdS gravity, with a suitable constant coefficient, doing the same regularizing job there.

Green-Schwarz Action and Chern-Simons Branes

Conjecture:

- The kinetic term of the Green-Schwarz action can be obtained from the transgression action boundary term, with the right relative coefficient
- Kappa symmetry is just what is left of gauge suspersymmetry after the dimensional reduction

CS BRANES & GREEN-SCHWARZ

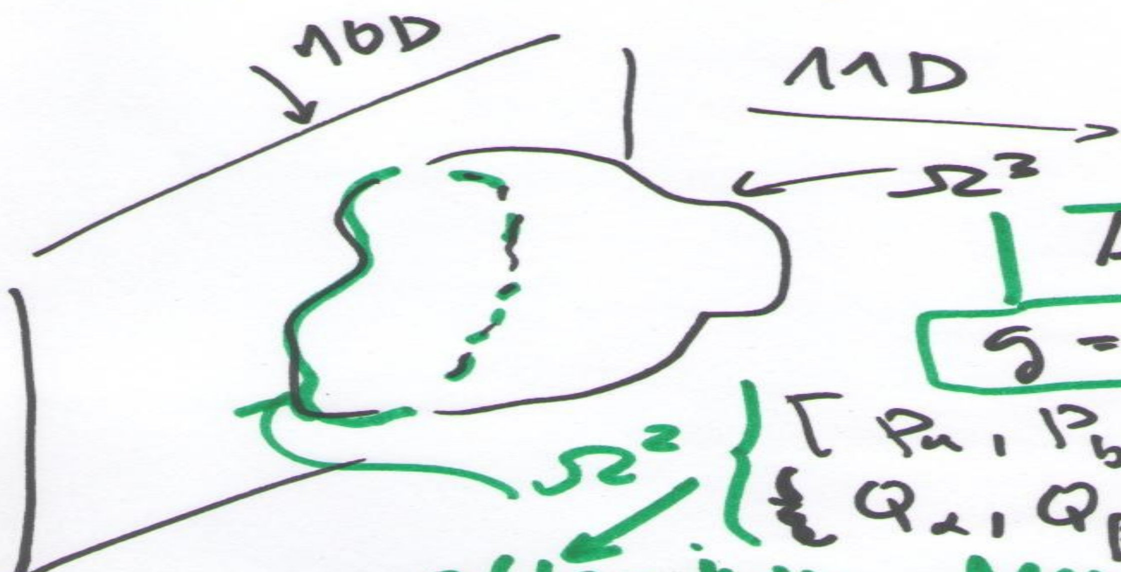
$OSp(32,1)$ $\{ P_a , Q_\alpha , M_{ab} , Z_{a_1 \dots a_5} \}$

P_a ↓ trans, Q_α ↓ SUSY, M_{ab} ↓ Lorentz

$\underline{d=10}$ $\langle T^a T^b \rangle \Leftarrow$ standard symmetric trace in the adjoint representation

$$S = \int_{\Sigma^2} d\sigma^2 \sqrt{-\gamma}\, \gamma^{ij} \langle J_i J_j \rangle + \frac{1}{2} \int_{\Sigma^3} \mathcal{Y}_3 (A_0, A_1)$$

$J = A_1 - A_0$ γ^{ij} auxiliary metric in the string world-sheet $\equiv$ boundary of the brane



$A_1 \equiv$ pure gauge

$A_0 = 0$

$$\boxed{A_1 = g^{-1} dg}$$

$$\boxed{g = e^{i(xP + \theta Q)}}$$

$$\begin{cases} [P_a , P_b] = 0 \\ \{ Q_\alpha , Q_\beta \} = 2i (C\gamma^a)_{\alpha\beta} P_a \end{cases}$$

after Inönü-Wigner contraction

Required traces to evaluate A_1 (non zero)

$\langle P_a P_b \rangle = M_{ab}$ $\langle P_a Q_\alpha Q_\beta \rangle = (\gamma_a C^{-1})_{\alpha\beta}$

CS BRANES & GREEN-SCHWARZ (cont.)

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$$e^{-M}\delta e^{M} = \delta M - \frac{1}{2}[M,\delta M] \text{ if}$$

$$[[M,\delta M],M] = [[M,\delta M],\delta M] = 0$$

$$\Rightarrow A_1 = i(dX_1^a - i\bar{\theta}_1\gamma^a\theta_1)P_a + d\theta_1^\alpha Q_\alpha$$

$$\Rightarrow \sqrt{-\gamma}\,\langle A_{0i}A_{0j}\gamma^{ij}\rangle = \sqrt{-\gamma}\,\eta_{ab}\Pi_i^a\Pi_j^b\gamma^{ij}$$

WZW term yield directly the right one after Inönü-Wigner contraction.

Conclusions

Why are CS branes different?:

Gauge invariance for space-time gauge group or supergroup
Background independence
Brane-background democracy
Quantization of constants and non renormalization
Already first quantized objects

Open problems and further directions of research:

Study of classical dynamics, initial value problem, static and non-static solutions, stability, etc.
Is there a consistency criterium to select the gauge group and space-time dimension, for instance anomalies, special algebraic structures (pure spinors, division algebras, triple algebras, twistors, etc.)?
In the line of the previous point, is there a more fundamental algebraic model where the differential structure gets dynamical content and the d, l and δ exterior derivatives of Mañés, Stora and Zumino are treated in the same footing?
Explicit solutions, to understand how this models work (as in recent work of Edelstein, Miskovic, Zanelli), and possible cosmological solutions, in the line of brane worlds, flux compactifications or Anabalon-Willison-Zanelli dimensional reductions, aiming at realistic cosmological models with GR plus dark energy in 4D
Non trivial topological phases and statistics when one swings around two of this objects, a la Wilczek,Wu, Zee and Zhang et al in higher dimensions.
Relationship with Bagger-Lambert-Gustavsson models?